FLUCTUATING HYDRODYNAMICS VIA HOLOGRAPHY.

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MOTIVATION :

- Understand long distance EFT/Hydrodynamics of (planar) holographic CFTs.
  - dual to black branes.
- The leading behaviour of the system is of dissipating excitations
  - dual to black brane Quasi-normal modes.
- Quantum mechanics predicts fluctuations / Noise [c.f. FDR]
  - black brane Hawking radiates.
- We will study the Fluctuating Hydrodynamics arising as these effects talk to each other.

- Example: BROWNIAN PARTICLE IN AIR.

  → The low energy effective theory consists of collective excitations of air like sound waves.
  → The particle 'senses' air molecules by exhibiting Brownian or stochastic evolution!

- Lesson: The Low energy EFT consists of long wavelength excitations of the medium & the stochastic evolution of probes coupled to the fast microscopic degrees of freedom.

- The low energy effective theory is an 'OPEN' theory obtained by integrating out fast modes.

- Our objective is to understand the evolution of the "reduced density matrix" of
  * probes coupled to fast or Markovian modes.
  * hydrodynamic modes — slow or non-Markovian.

Q 1: How do we study such OPEN field theories ?

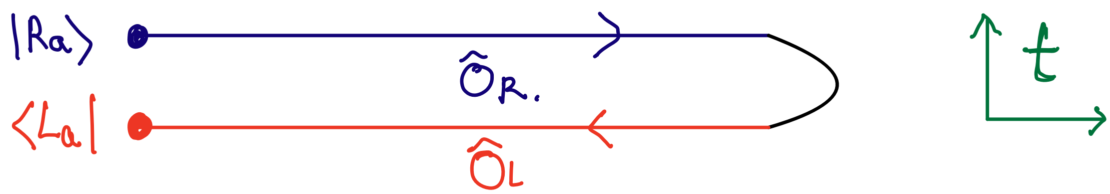
Q2 : Given a holographic model,

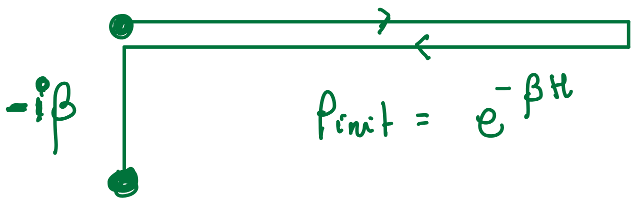
- How do we separate out fast-slow dynamics.
- How do we selectively integrate out fast modes, while retaining the slow modes in the effective theory.

SCHWINGER-KELDYSH FORMALISM (Lightning Review).

- Start with initial density matrix $\rho_{init} = \sum_a C_a |R_a\rangle\langle L_a|$
- S-K generating functional Z_{SK}.

$$Z_{SK}[J_R^{\hat{O}}, J_L^{\hat{O}}] \equiv \mathrm{Tr}\left\{ e^{iJ_R^{\hat{O}}\hat{O}} \rho_{init}\, e^{-iJ_L^{\hat{O}}\hat{O}} \right\}$$





* Doubling of field contents

$$\hat{\phi} \to \hat{\phi}_R \otimes \hat{\phi}_L$$

* Contour ordered correlators.

R fields are time ordered.

L fields are anti-time ordered.

* R–L identification at future most time ~ tracing.

* $Z_{SK}[J_R = J_L] = 1$: Collapse Rules.

*

$$S_{SK}[J_R, J_L] = S_0[J_R] - S_0[J_L] + S_{IF}[J_R, J_L]$$

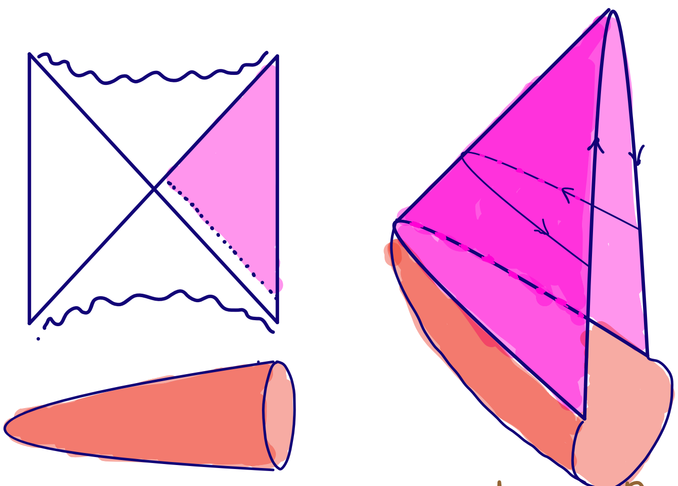
INFLUENCE FUNCTIONAL.

Gravitational Schwinger-Keldysh Saddle:

* CFT on manifold M

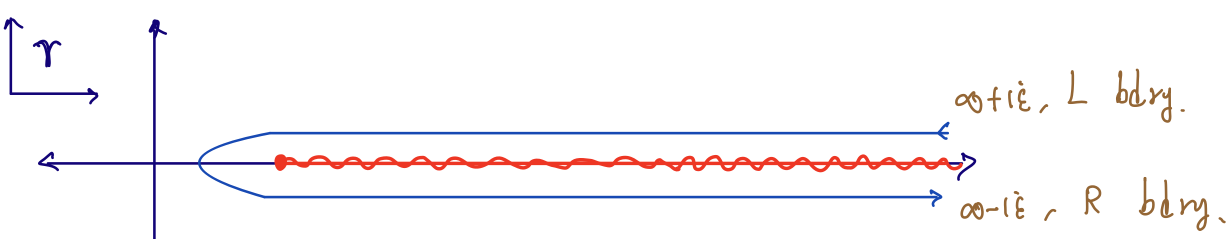
  $=$ Gravity on B with $\partial B = M$.

* We need a manifold with doubled boundary.



* Avoids issues with singularity
* Only asymptotic boundaries
* What is the gluing condition at the horizon?

* Crossley - Glorioso - Liu Prescription   1812.08785



Radial contour at constant ingoing time v

* Demand analyticity in radial co-ordinate in ingoing Eddington-Finkelstein co-ordinates.

* Modus operandi: Exploit time-reversal isometry.
    - Derive ingoing solution (analytic).
    - Time-reverse using $\mathbb{Z}_2$ involution isometry

$$v \mapsto -v + i\beta\zeta \qquad\qquad i\beta\zeta \sim 2r^*$$

- $$\Phi_{SK} = J_{\bar{F}}\, G^{In}(\omega,k) + J_{\bar{\phi}}\, e^{-\beta\omega\zeta}\, G^{In}(-\omega,k)$$

$$J_{\bar{F}} = (n_B+1) J_R - n_B J_L \quad ; \quad J_{\bar{\phi}} = n_B (J_R - J_L)$$

$$n_B = \frac{1}{e^{\beta\omega} - 1}$$

CONSISTENCY CHECKS :

* Schwinger-Keldysh collapse rule + KMS conditions.

    SATISFIED from general analyticity arguments.

* Correct counterterm structure. SATISFIED

$$S_{ct} = S_{ct}[R] - S_{ct}[L]$$

* Fermions; Charged & Rotating branes; Interactions.

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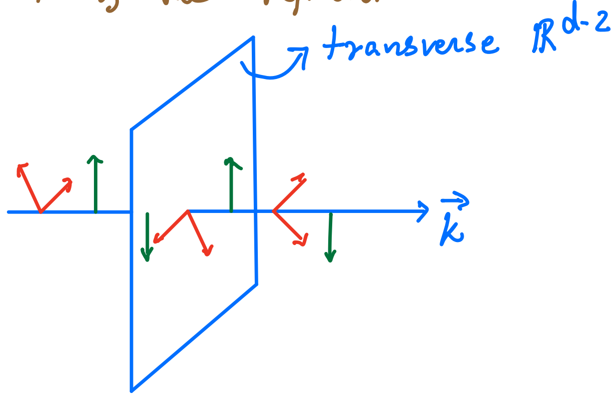
DISTILLING OUT THE FLUID MODES.

Boundary Perspective:

* We need a general procedure to separate out Markovian & non-Markovian dynamics
* Let us focus on a d-dimensional CFT.
  Working principle: Implement $SO(d-2)$ harmonic decomposition of excitations.

* Hydrodynamic excitations characterised by the wave vector $\vec{k}$ & the representation on the transverse $SO(d-2)$.



$\frac{d(d-3)}{2}$ Spin-2 : $\mathbb{T}_{ij}$

$d-2$ Spin-1 : $\mathbb{V}_i$

Examples.

a) $U(1)$ current J_μ : Dual to Bulk Maxwell theory.

$$J_i = J_S\, \partial_i \mathbb{S} + J_V\, \mathbb{V}_i$$

J_V : trivially conserved! Dual to radially infalling photons – Markovian.

J_S : charge diffusion. Dual to photons grazing the black brane – Non-Markovian.

b) Stress Tensor $T_{\mu\nu}$: Dual to bulk metric fluctuations.

$$T_{ij} = T_S\, \partial_i \partial_j \mathbb{S} + T_V\, \partial_{(i} \mathbb{V}_{j)} + T_T\, \mathbb{T}_{ij}$$

T_T : trivially conserved! Radially infalling gravitons.

T_V : transverse momentum diffusion

Gravitons graze the black brane

T_S : Energy/pressure waves : sound mode.

- At the quadratic order $SO(d-2)$ decomposition separates out the d.o.f with different behaviours.

Bulk Perspective : Designer Scalars.

- To understand how different kinds of excitations behave in the bulk, we start with designer scalar actions with general dialaton coupling.

- $S[\phi_M] = \int_{bulk} \sqrt{-g}\, e^{\chi_s}\, \partial\phi_M \partial\phi_M$; $e^{\chi_s} = r^{M-d+1}$

- e^{χ_s} selectively amplifies coupling in the bulk.

- $M+1 > 0$ is Markovian, else non-Markovian.

- The d.o.f in the diffusing sectors of Maxwell & metric perturbations can be mapped to these scalars!

- $J_V \rightarrow M = d-3$, $\quad J_S : M = 3-d$. [Kodama-Ishibashi]

  $T_T \rightarrow M = d-1$, $\quad T_V : M = 1-d$

$$A_\mu dx^\mu = \frac{\mathbb{S}}{r^{d-3}}\left[dv\, \mathbb{D}_+ - dr\frac{d}{dr}\right]\phi_{3-d} + \phi_{d-3}\, \mathbb{V}_i\, dx^i$$

$$\delta ds^2 = \mathbb{S}(\ldots) + r^2\left\{ 2\,\frac{\mathbb{V}_i dx^i}{r^{d-1}}\left[dv\, \mathbb{D}_+ - dr\frac{d}{dr}\right]\phi_{1-d} + \phi_{d-1}\, \mathbb{T}_{ij}\, dx^i dx^j \right\}$$

$$\mathbb{D}_+ \equiv r^2 f\partial_r + \partial_v \sim \left(\frac{\partial}{\partial r}\right)_{t_{schw}}$$

- Markovian Sector:
  - → All ingoing modes are non-normalisable.
  - → Ingoing modes produced by sources quickly dissipate.
  - → Only Hawking modes are normalisable.
  - → Boundary value of ϕ_M is the source & its conjugate is the vev.
- Non-Markovian Sector:
  - → "Some" ingoin modes are normalisable.
  - → These modes satisfy the diffusive dispersion & out-live their sources.
  - → Boundary value of ϕ_M is the vev & its conjugate is the source.
- Conjugate of ϕ_M is always derivative expandable. in terms of its boundary value
  - → In Markovian Case, vev is a local function of source. There are no long distance correlations
  - → For non-Markovian case, the source is a local function of vev. This gives the dynamical equation for the vev.
- Ts, the scalar sector of gravity has a slightly. more complicated scalar description.

- In terms of original metric/Maxwell fields:
  - → Dirichlet/Source boundary condition for Markovian
  - → Neumann/vev boundary condition for non-Markovian
- The standard AdS/CFT dictionary with Dirichlet boundary conditions yields generating functionals.
- Moving to Neumann conditions requires additional boundary terms ≡ Legendre transformation.
  - → This is in built in the scalar formalism.
- Wilsonian Influence Functional:

$$\mathbb{W}_{IF}[\Phi_{\text{non-Mar}}, J_{\text{Mar}}]$$

$$\simeq \mathbb{Z}_{SK}[J_{\text{non-Mar}}, J_{\text{Mar}}] - J_{\text{non-Mar}}\,\Phi_{\text{non-Mar}}$$

$$\Phi_{\text{non-Mar}} \sim \frac{\delta\, \mathbb{Z}_{SK}}{\delta J_{\text{non-Mar}}}.$$

CONCLUSION & DIRECTIONS.

- We motivated a formalism for distilling out modes with different hydrodynamic behaviour
- We showed a gauge invariant formalism to deal with bulk gauge field excitations.
- Gravitational S-K saddle → derivation ?
  → generalising beyond equilibrium
- checking gravitational interactions?

THANK YOU !! :)