Breakdown of hydrodynamics from holographic pole collision

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finished with Yan Liu, based on
arXiv: 2111.07770

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Outline

01 Backgrounds

02 Motivation & The model

03 Results

04 Further questions & discussion

Hydrodynamics

- hydrodynamics focus on long-distance, late-time macroscopic process

$$T, \mu, u^{\nu} \dots$$

- dynamics are described by conserved equations

$$\partial_\mu T^{\mu\nu} = 0\,. \qquad \partial_\mu J^{\mu} = 0\,.$$

- series expansion

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + p\Delta^{\mu\nu} - \eta\,\Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{d}\eta_{\alpha\beta}\partial_\mu u^\mu\right) - \zeta\Delta^{\mu\nu}\partial_\lambda u^\lambda + O(\partial^2)\,,$$

$$J^\mu = n u^\mu - \sigma T\Delta^{\mu\nu}\partial_\nu(\mu/T) + \chi_{\mathrm{T}}\Delta^{\mu\nu}\partial_\nu T + O(\partial^2)\,.$$

does it converge or not?

- susceptibility, dc transport

$$D\chi = -\lim_{\omega\to 0}\lim_{k\to 0}\frac{\omega}{\mathbf{k}^2}\mathrm{Im}\,G^R_{nn}(\omega,\mathbf{k})\,.$$

$$\sigma \sim \lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\;G^R_{J_xJ_x}(\omega, k=0) \qquad \sigma \sim \lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\;G^R_{J_xJ_x}(\omega, k=0)$$

- hydrodynamic modes(e.g diffusive mode)

$$\omega_{diff} = -iD_c k^2 + \dots = -i\sum_{n=1}^{\infty} c_n (k^2)^n$$

Kovtun, 1205.5040

Complex spectral curve

- go to complex ω, k plane

$$P(k^2,\omega)=0 \;\; \rightarrow \;\; \omega(k^2) \;, \omega, k \in \mathbb{C}$$

- hydrodynamic series are Puiseux series around a regular critical point

$$P(q_*^2,\omega_*)=0,\; \partial_{\omega_*}P(k_*^2,\omega_*)=0,\; \ldots \partial_\omega^p P(k_*^2,\omega_*)\neq 0$$

- charge diffusive series is a Tylor series around $(k^2,\omega)^{p=1,regular}_{diffusive}=(0,\,0)$

$$\omega_{diff}=-i\sum_{n=1}^{\infty}c_n(k^2)^n=-iD_ck^2+\ldots$$

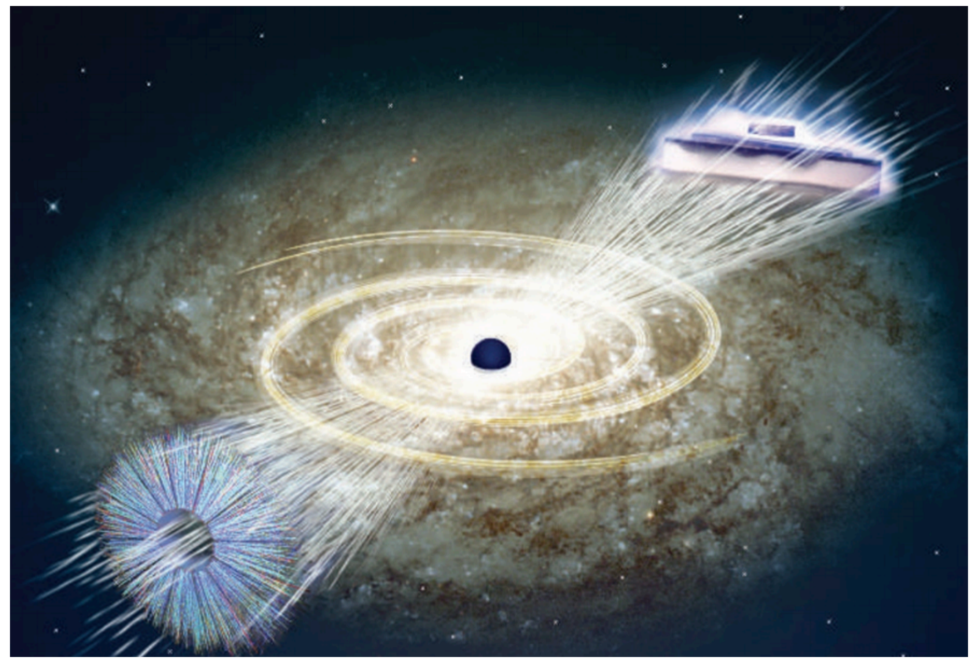
- this series converges until we find the next critical point, that is pole collision

Grozdanov, Kovtun, Starinets, Tadic 1904.01018 & 1904.12862

Holography

- strongly-coupled liquid = weak, classical gravity

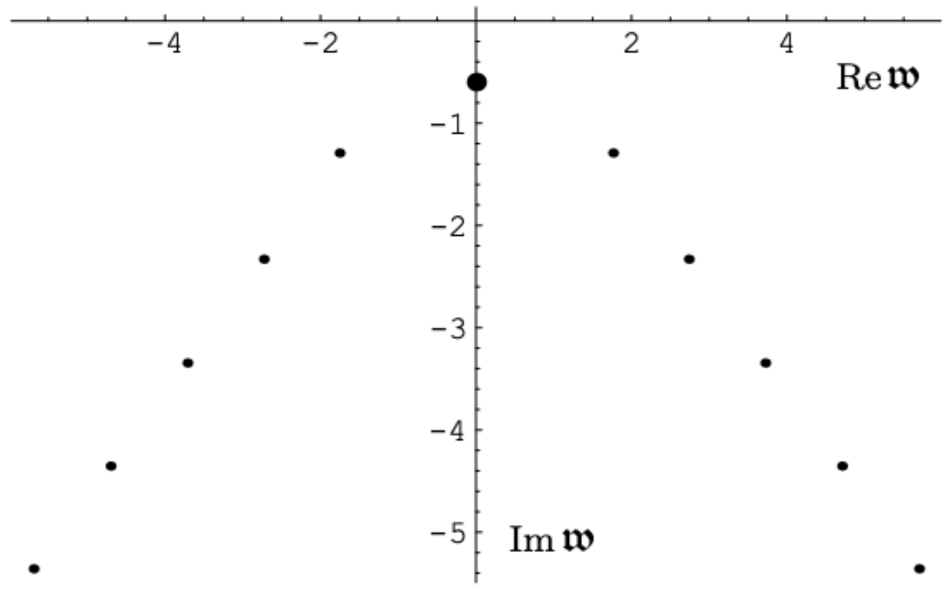
large N 1 dimensional higher



- spectrum of excitations in this liquid = quasi-normal modes of black hole

$$G^R(\omega, k) = \frac{B(\omega, k)}{A(\omega, k)}$$

$A(\omega, k) = 0 \rightarrow$ hydrodynamic mode, gapped mode

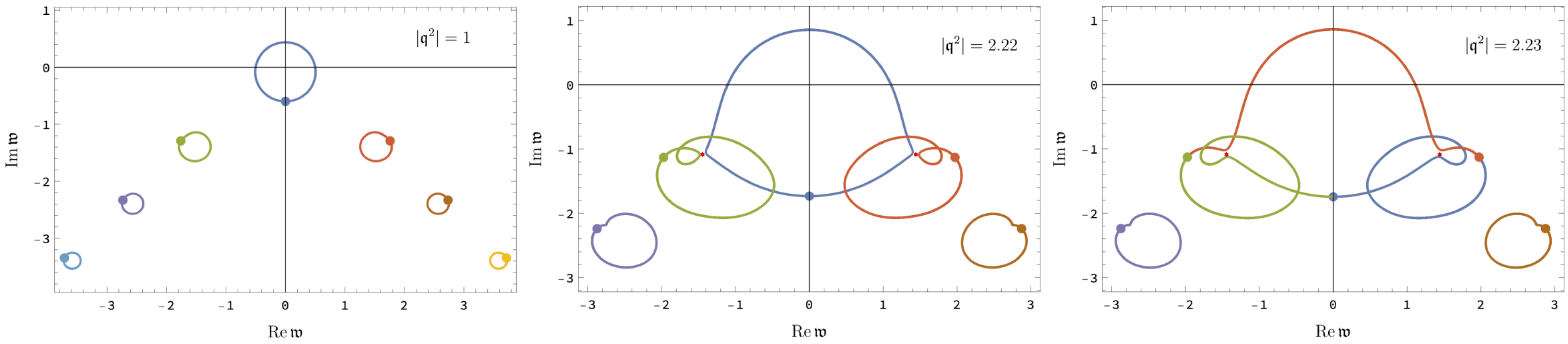


Kovtun, Starinets hep-th0506184

Complex spectral curve

- an example of shear mode

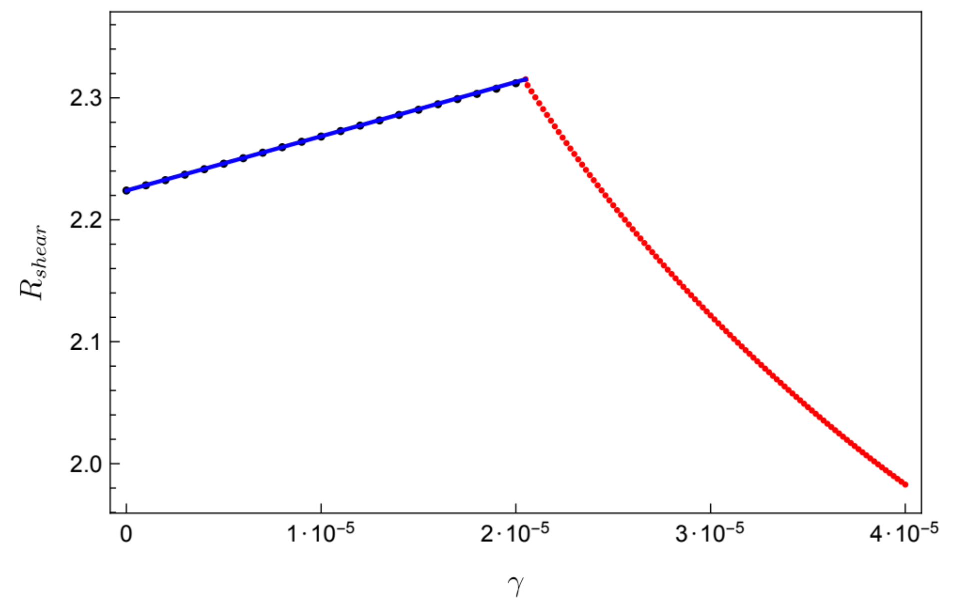
$$q = |q| e^{i\varphi}, \phi \in [0,\pi)$$



Grozdanov, Kovtun, Starinets, Tadic 1904.01018

Pole collision: applications

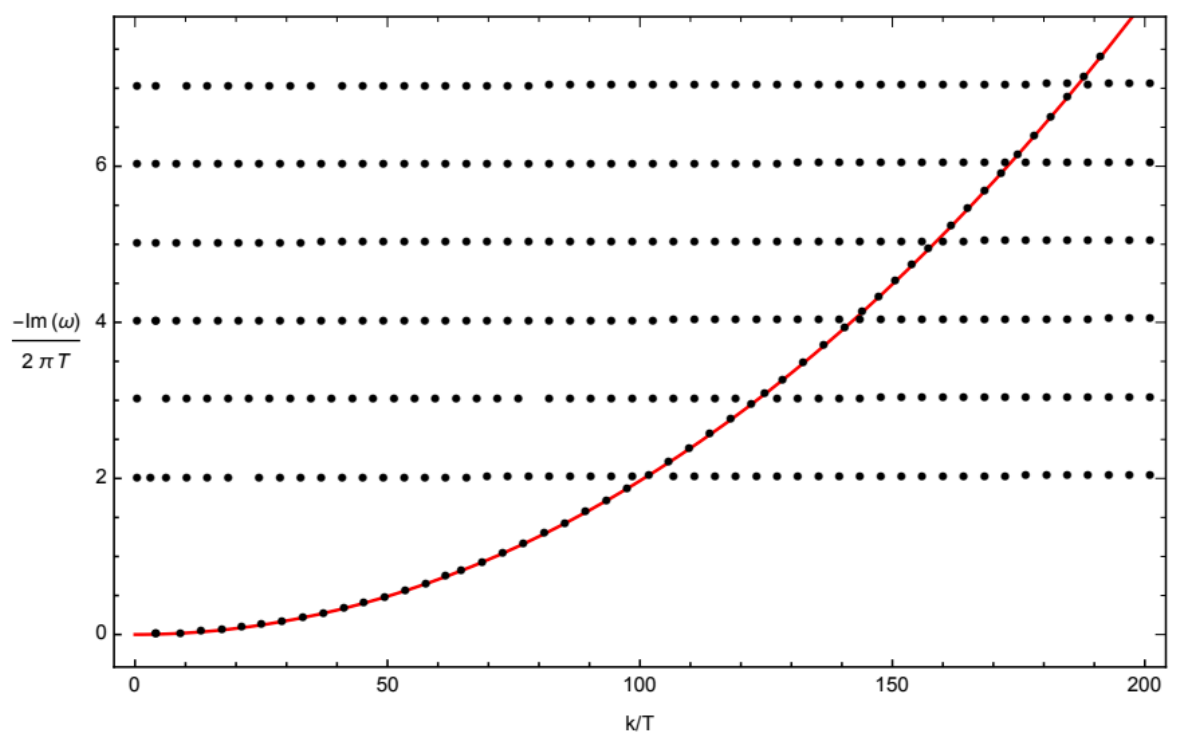
- the stronger the coupling, the wider the domain of hydrodynamics?



$\gamma \propto \lambda^{-3/2}$

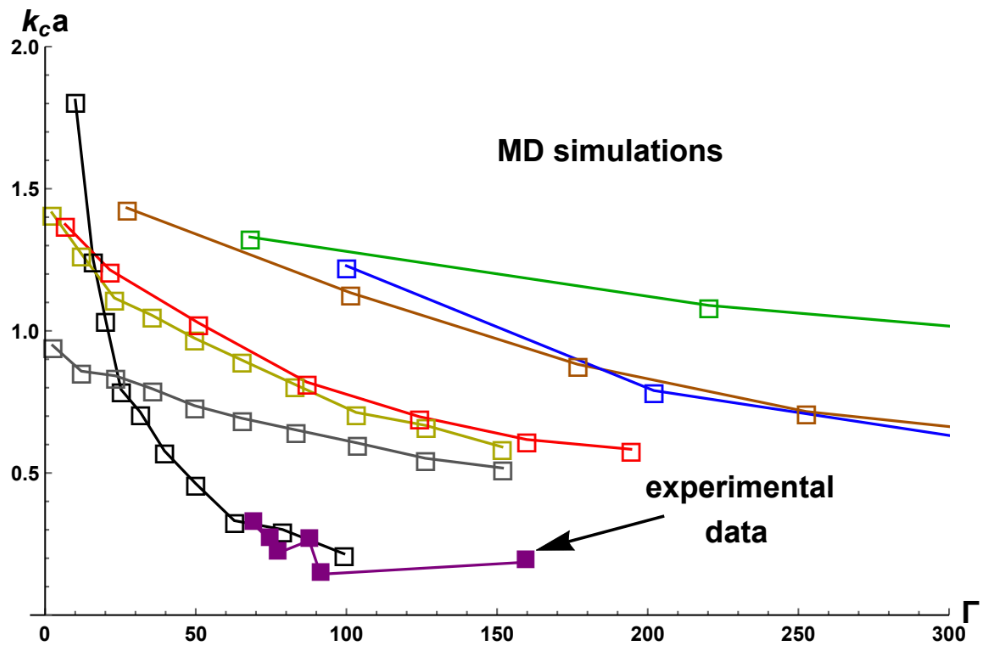
λ the 't Hooft coupling

- $\omega \sim k \ll T$?



$$\frac{k}{T} \sim O(10^2)$$

- what coupling shall we use when we say
  "hydrodynamics works well for strongly-coupled liquid"?



$$U(r) = \frac{Q}{4\pi\epsilon_0 r} e^{-r/\lambda_D}$$

$$\Gamma \equiv \frac{Q^2}{4\pi\epsilon_0 a k_B T}$$

the strong coupling is not the coupling for Coulomb repulsion

Grozdanov, Starinets, Tadic 2104.11035;

Baggioli, 2010.05916;

Arean, Davison, Gouteraux, Suzuki, 2011.12301

Diffusion bound

- KSS bound is a transport bound, and can be rewritten as a diffusion (lower) bound

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B} \qquad\qquad D_{shear} \geq \frac{c^2}{4\pi}\tau_{pl}$$

- It is natural to expect diffusion of conserved quantities has a lower bound and an upper bound

$$\boxed{v_?^2 \tau_? \lesssim D \lesssim v_{??}^2 \tau_{??}}$$

Published: 23 December 2014

Theory of universal incoherent metallic transport

Sean A. Hartnoll

Nature Physics **11**, 54–61 (2015) | Cite this article

2803 Accesses | **219** Citations | **32** Altmetric | Metrics

Editors' Suggestion

Upper Bound on Diffusivity

Thomas Hartman, Sean A. Hartnoll, and Raghu Mahajan

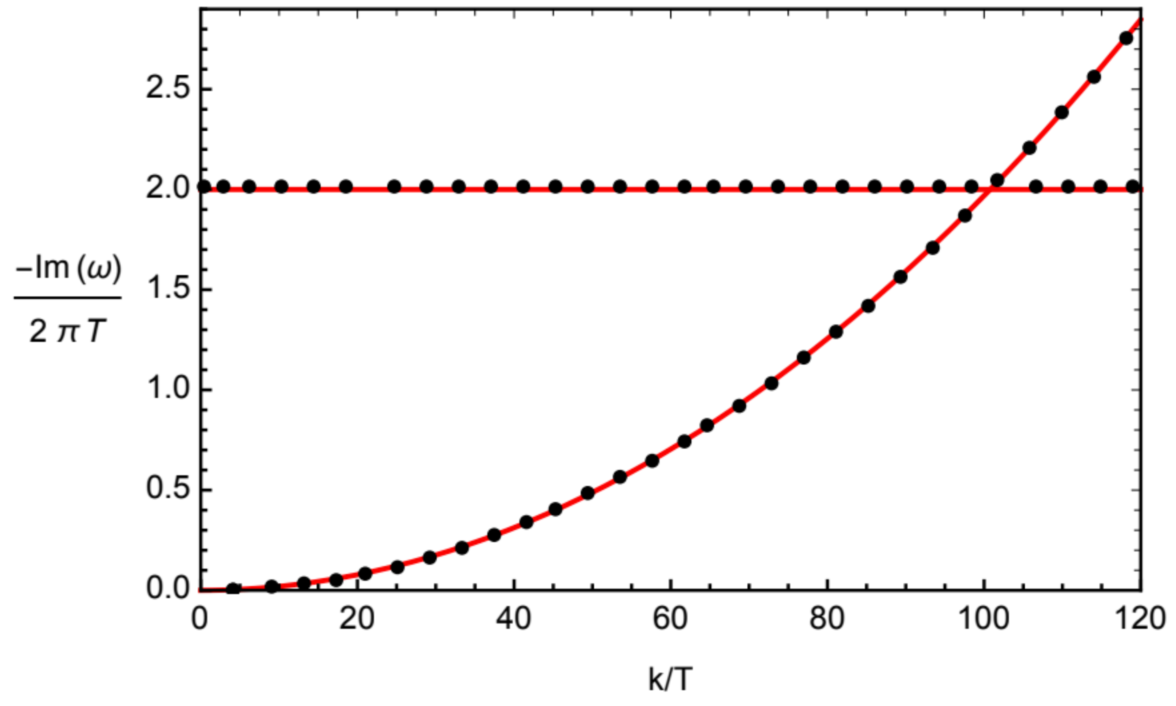
Phys. Rev. Lett. **119**, 141601 – Published 2 October 2017

Diffusion upper bound

- recently, an upper bound based on equilibrium velocity v_{eq}, equilibrium time τ_{eq} is proposed, and v_{eq} and τ_{eq} are defined from convergence of hydrodynamics.

$$v_{eq} \equiv \frac{\omega_{eq}}{k_{eq}}, \quad \tau_{eq} \equiv \frac{1}{\omega_{eq}}, \quad D \to v_{eq}^2 \tau_{eq} \quad \text{as} \quad T \to 0$$

- they find that, pole collision occurs between hydrodynamic mode and lowest IR mode, at complex momentum



Arean, Davison, Gouteraux, Suzuki, 2011.12301

Jeong, Kim, Sun, 2105.03882,

Wu, Baggioli, Li, 2102.05810

Our motivation

- conformal to $AdS_2 \times R^2$ IR geometry, does the IR mode picture still exist?

  this is a different fixed point from $AdS_2 \times R^2$, ground state is stable

- consider charge diffusion

  for a neutral strongly-coupled system, charge diffuses independently with energy,

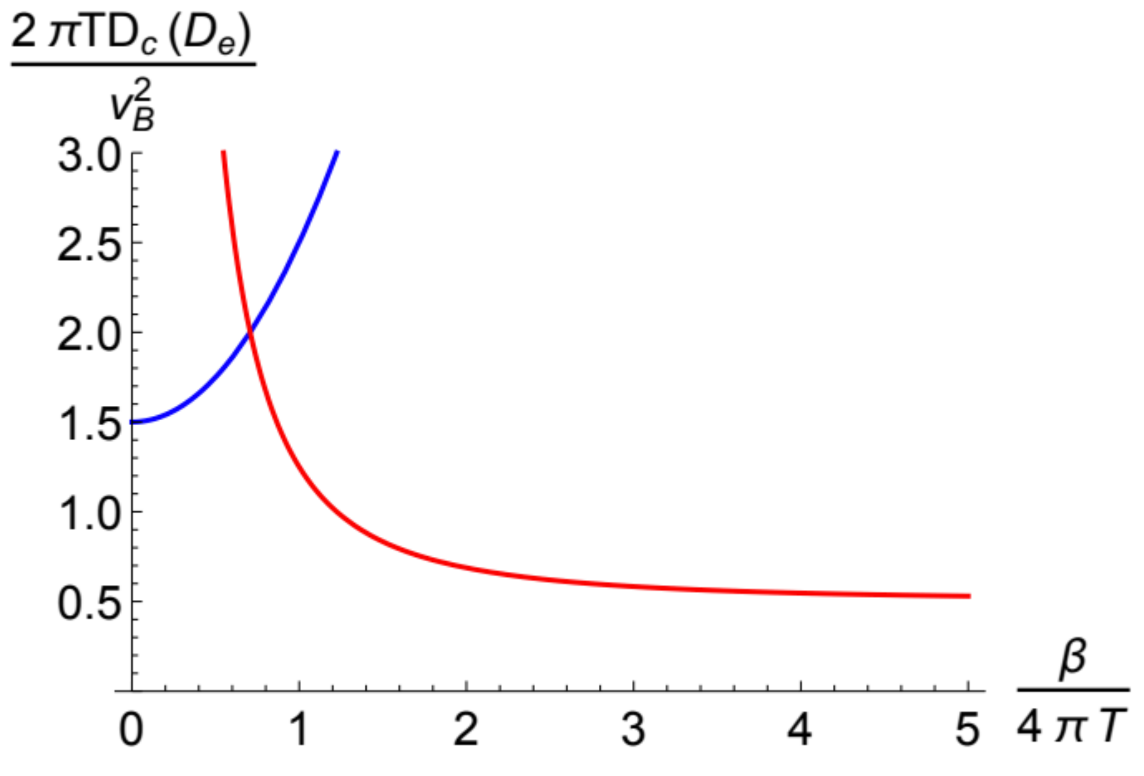
  it should have its own equilibrium property, e.g $\tau_{eq}, v_{eq} \dots$

- charge diffusion bound

  at low temperature, D_c does not saturate the lower

  bound defined from τ_L, v_B, and seems to diverge within

  this definition.



Kim, Niu 1704.00947

The model

- Gubser-Rocha model with linear axions

$$S = \int d^4x\sqrt{-g}\left(R - \frac{1}{4}e^{\alpha\phi}F^2 - \frac{3}{2}(\partial\phi)^2 + \frac{6}{L^2}\cosh\phi - \frac{1}{2}\sum_{I=1}^{2}(\partial\psi_I)^2\right)$$

- dilation with special $V(\phi)$ + axions: induce an RG-flow to an IR fixed point at $T = 0$

conformal to $AdS_2 \times R^2$

- properties

$s \propto T$ at low T, stable ground state

$\rho \propto T$, linear in T resistivity when $\alpha = 1$, similar to strange metals (axions or graviton mass)

- effective gauge coupling, α is real and tunable

$$\boxed{g_{eff}^2 = e^{-\alpha\phi}}$$

The model

$\boxed{A=0}$

- solution

g_{tt} $$u = \sqrt{r}(r-r_0)\left(r-r_0+\sqrt{2}m\right)\frac{1}{\sqrt{r-r_0+\frac{m}{\sqrt{2}}}},$$

g_{ii} $$f = \sqrt{r}\left(r-r_0+\frac{m}{\sqrt{2}}\right)^{3/2},$$

$$\phi = \frac{1}{2}\log\left(\frac{r-r_0+\frac{m}{\sqrt{2}}}{r}\right),$$

$$\psi_I = m x_I\,.$$

- $T\to 0$ limit, IR geometry under $\boxed{r=\frac{\sqrt{2}}{m\zeta^2}}$

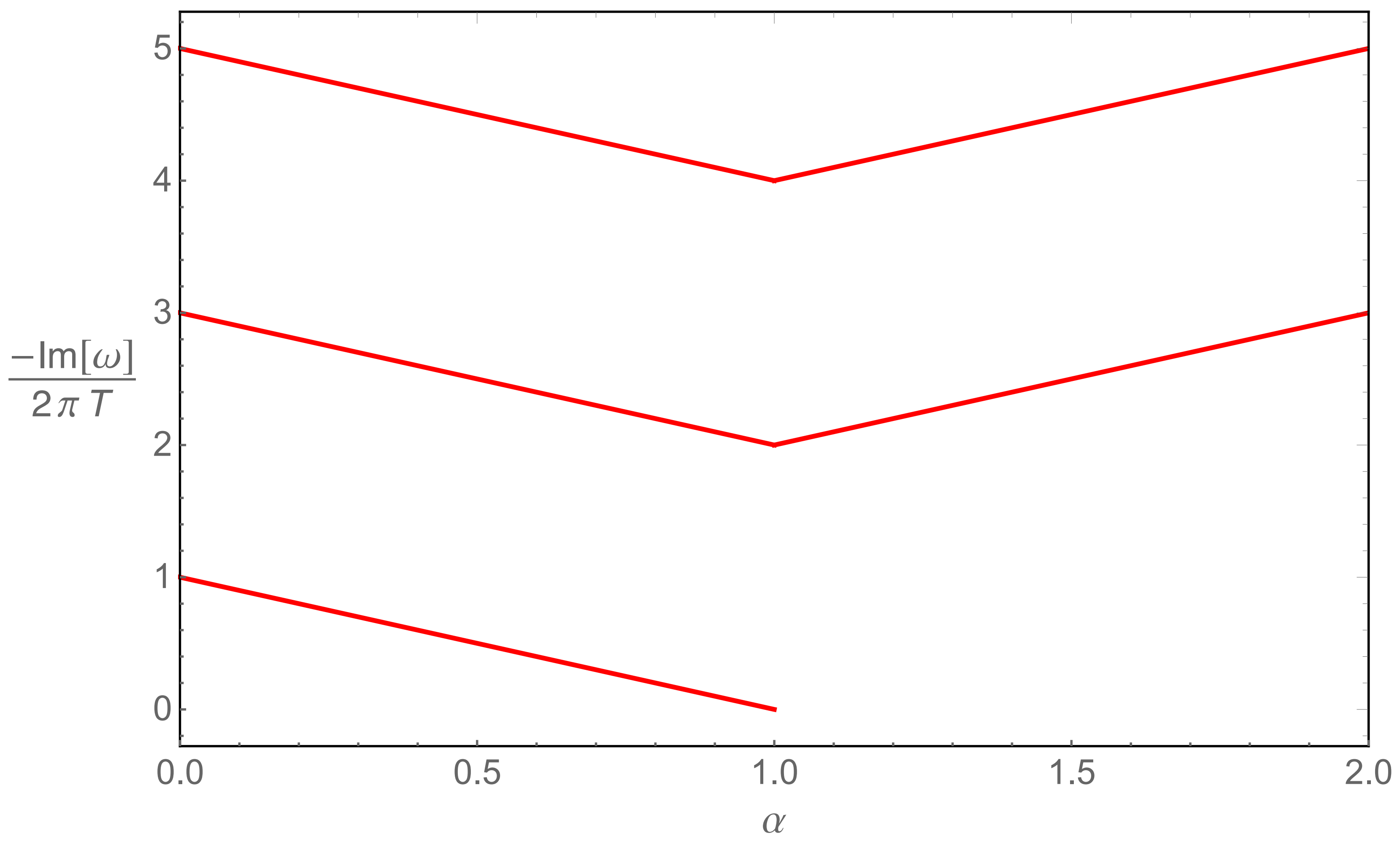
$$ds^2 = \frac{2\sqrt{2}}{m\zeta}\left(\frac{1}{\zeta^2}\left(-dt^2+d\zeta^2\right)+\frac{m^2}{4}\left(dx^2+dy^2\right)\right)$$

$$(t,\ \zeta,\ x,\ y)\to(\lambda t,\ \lambda\zeta,\ x,\ y) \qquad ds^2\to\lambda^{-1}ds^2$$

semi-local quantum liquid

- IR poles at k=0

  - When $\alpha<1$, the IR poles are located at $\frac{i\omega}{2\pi T}=2n-1-\alpha$ with positive integer $n=1,2,\ldots.$
  - When $\alpha>1$, the IR poles are located at $\frac{i\omega}{2\pi T}=2n-1+\alpha$ with positive integer $n=1,2,\ldots.$



Results

case 1: hydro mode collides with a slow mode

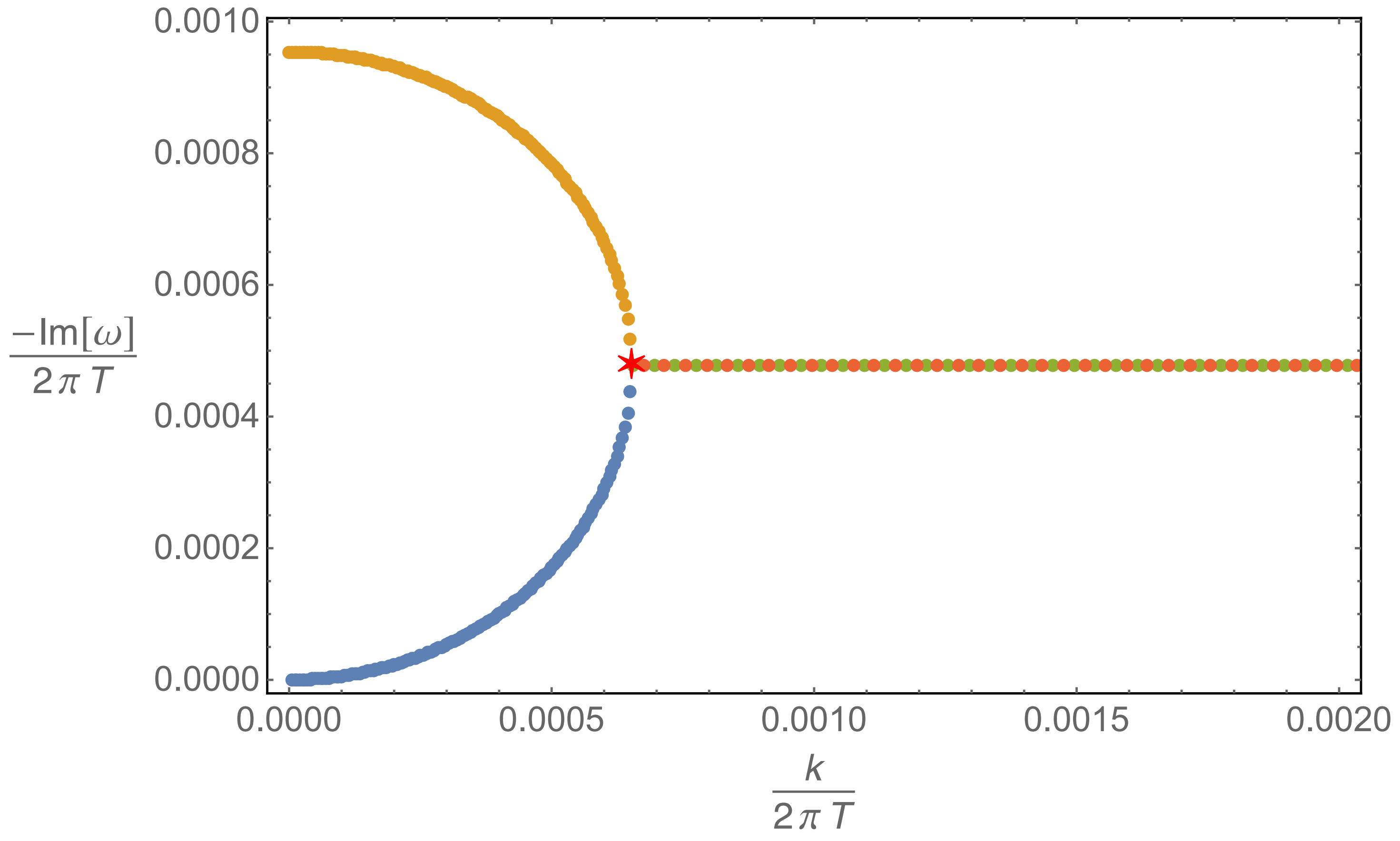
case 2: hydro mode collides with an IR mode

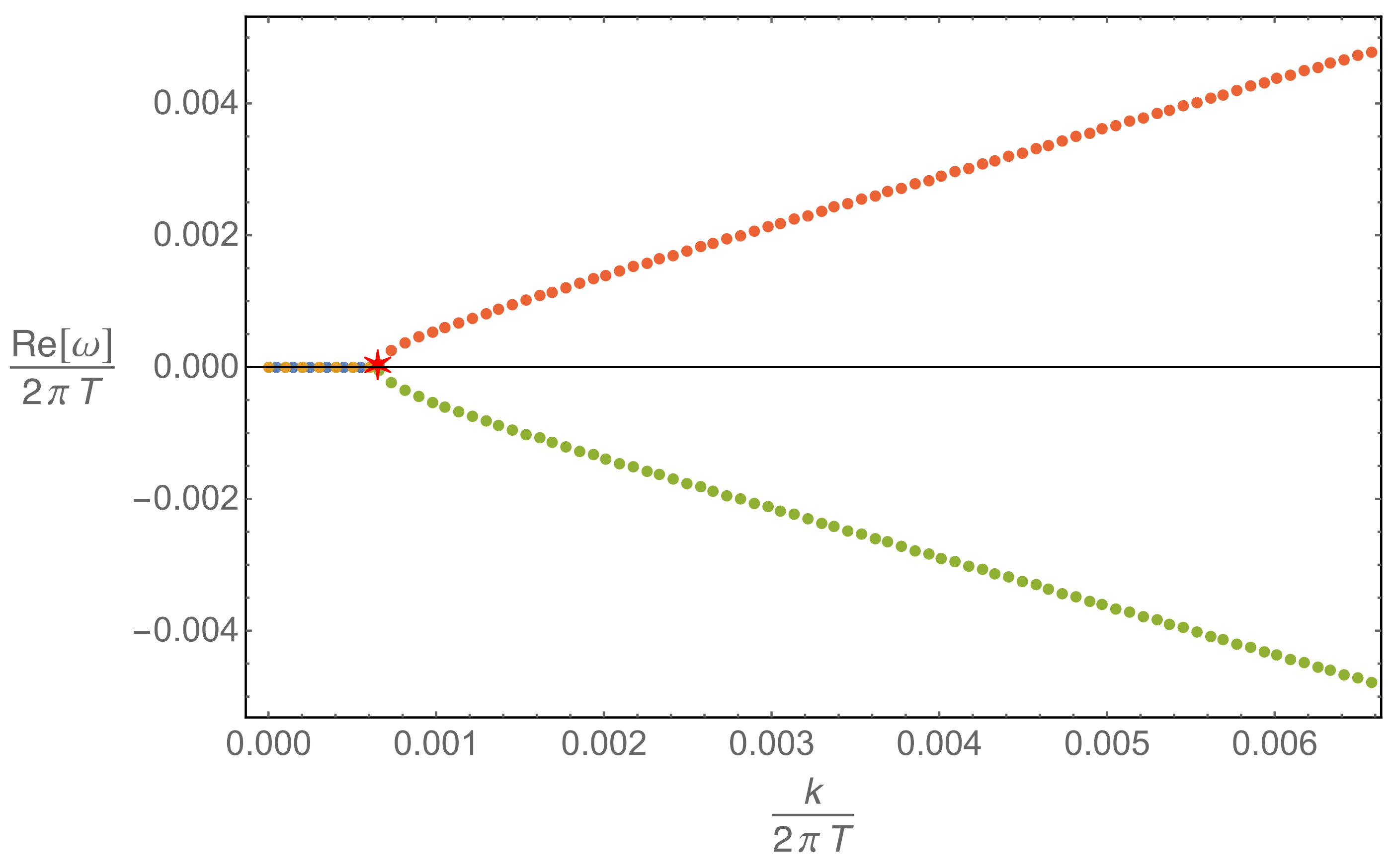
general cases

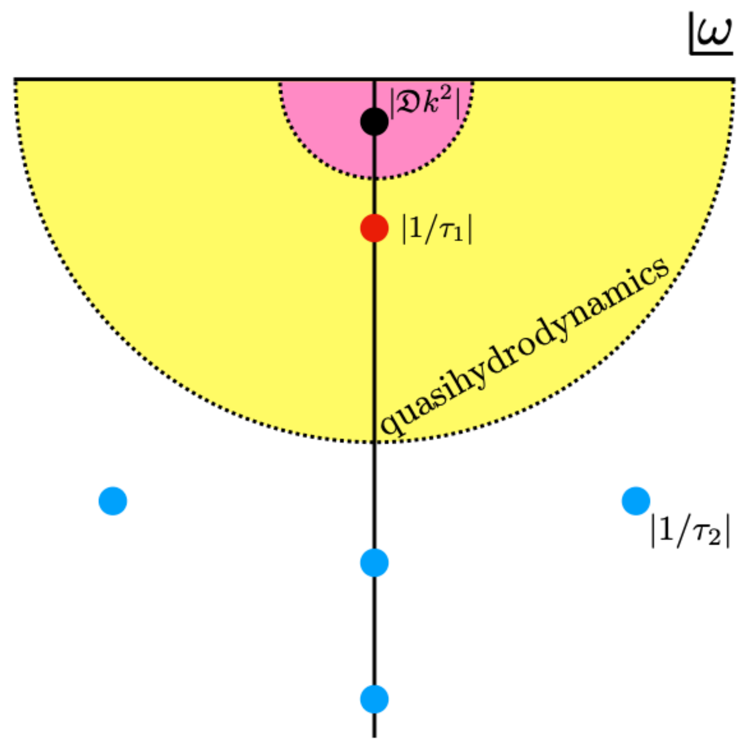
Results - slow mode

At $\alpha = 2$, the first non-hydro mode is a slow mode, it collides with charge diffusive mode

- collision occurs between a *slow* mode and the *diffusive* mode $\quad T \to 0$







- slow mode: $T\tau \gg 1$

quasi-hydrodynamic picture

- pole collision locates at real $k = k_{eq}$ and pure imaginary $\omega = -i\omega_{eq}$

Grozdanov, Lucas, Poovuttikul, 1810.10016

Results - slow mode

The first non-hydro mode is a slow mode, it collides with charge diffusive mode

$$\tau = \int_{r_0}^{\infty} dr \left(\frac{e^{\alpha\phi_0}}{u e^{\alpha\phi}} - \frac{1}{4\pi T}\frac{1}{r-r_0} \right), \qquad D_c = \int_{r_0}^{\infty} dr\, \frac{e^{\alpha\phi_0}}{f e^{\alpha\phi}}$$

- telegrapher equation

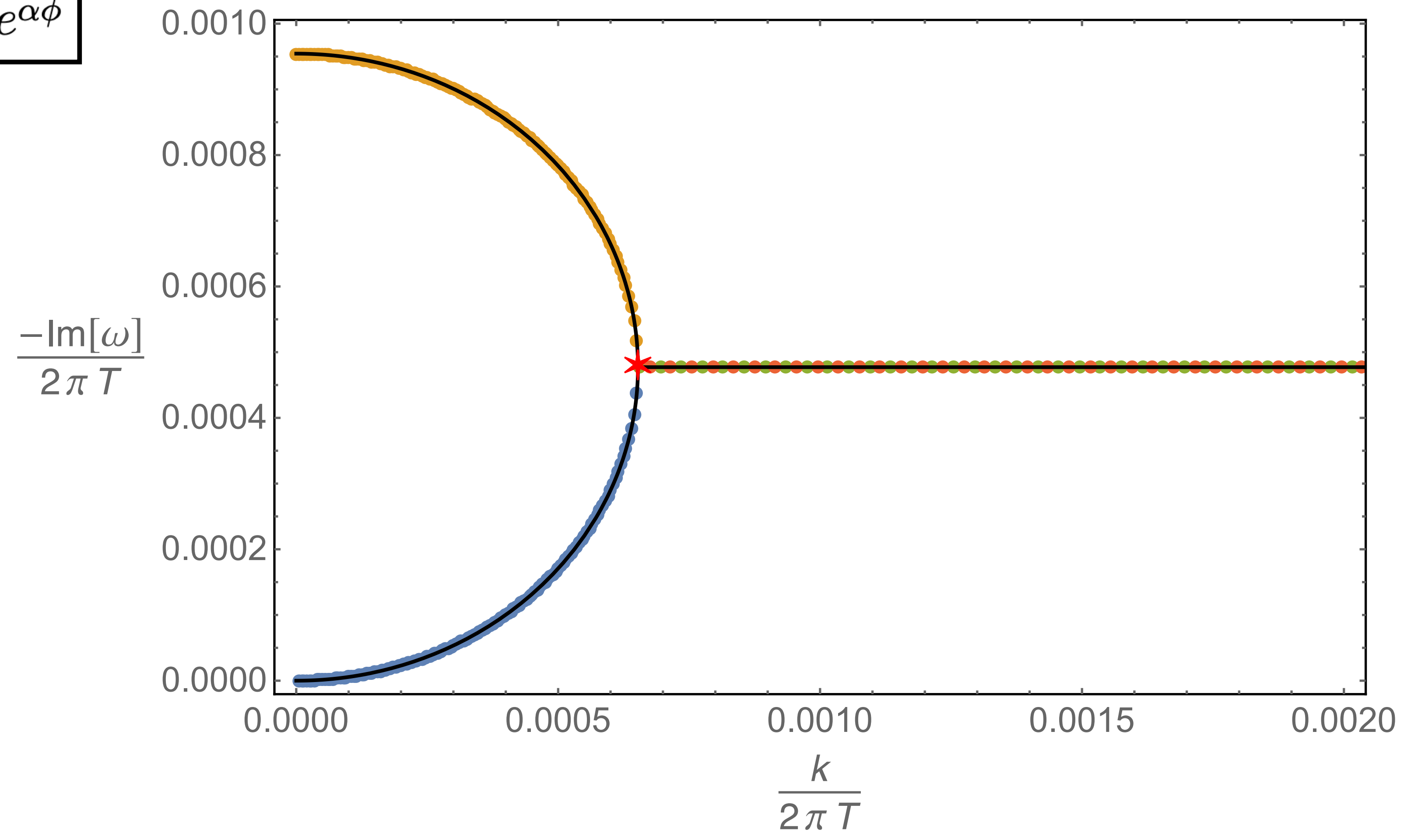
$$\omega^2 + \frac{i}{\tau}\omega - \frac{D_c}{\tau}k^2 = 0$$

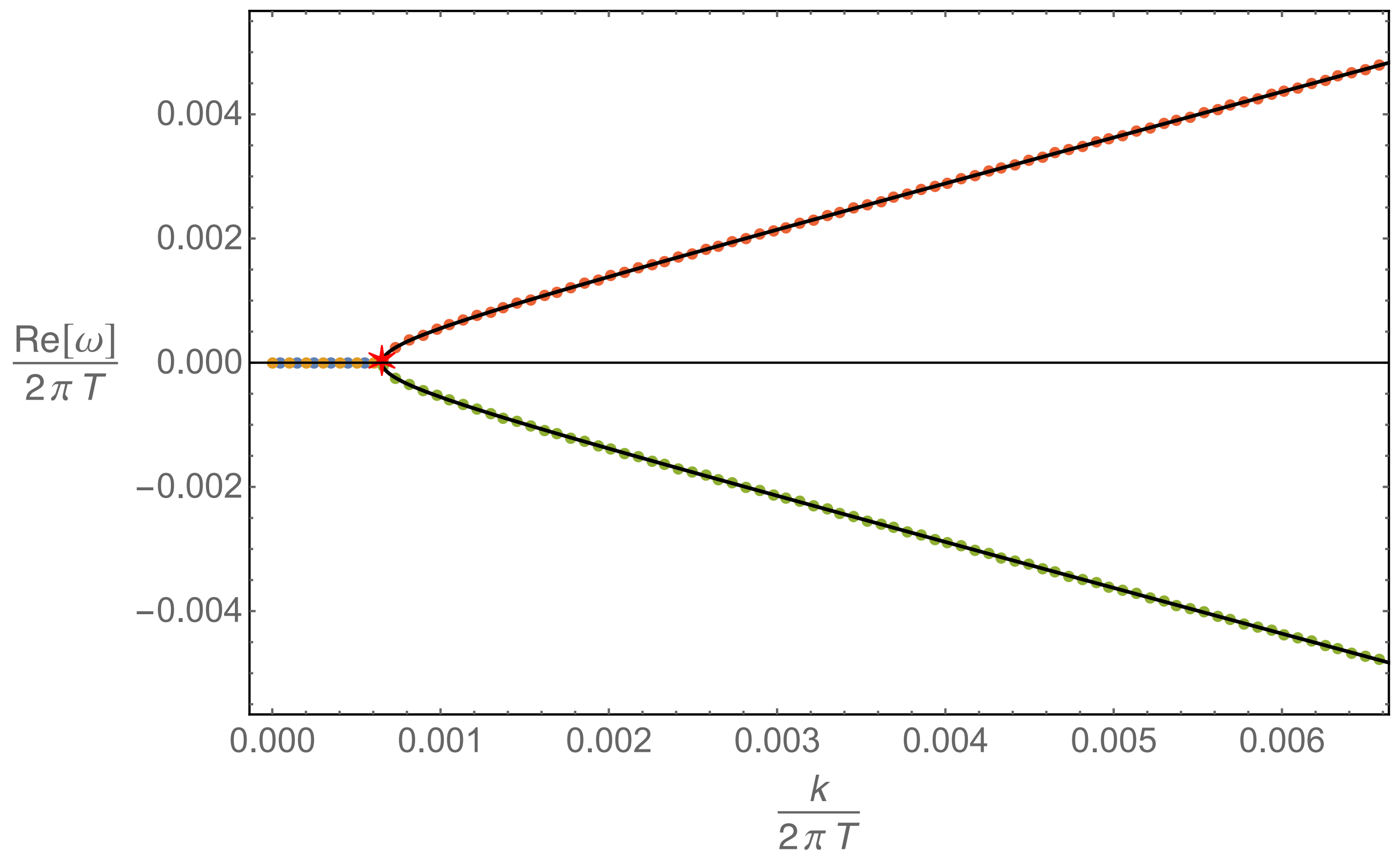
- dispersion relation

$$\omega_\pm = -\frac{i}{2\tau}\left(1 \pm \sqrt{1 - 4D_c\tau k^2}\right)$$

- location of collision

$$(k_{eq}\,,\ \omega_{eq}) = \left(\frac{1}{\sqrt{4D_c\tau}}\,,\ \frac{1}{2\tau} \right)$$





Results - slow mode

The first non-hydro mode is a slow mode, it collides with charge diffusive mode

- breakdown of hydrodynamics (low temperature)

$$\boxed{(k_{eq}\,,\ \omega_{eq}) = \left(\frac{1}{\sqrt{4D_c\tau}}\,,\ \frac{1}{2\tau}\right)}$$

$$k_{eq} \sim \frac{T^2}{m}, \omega_{eq} \sim \frac{T^2}{m}$$

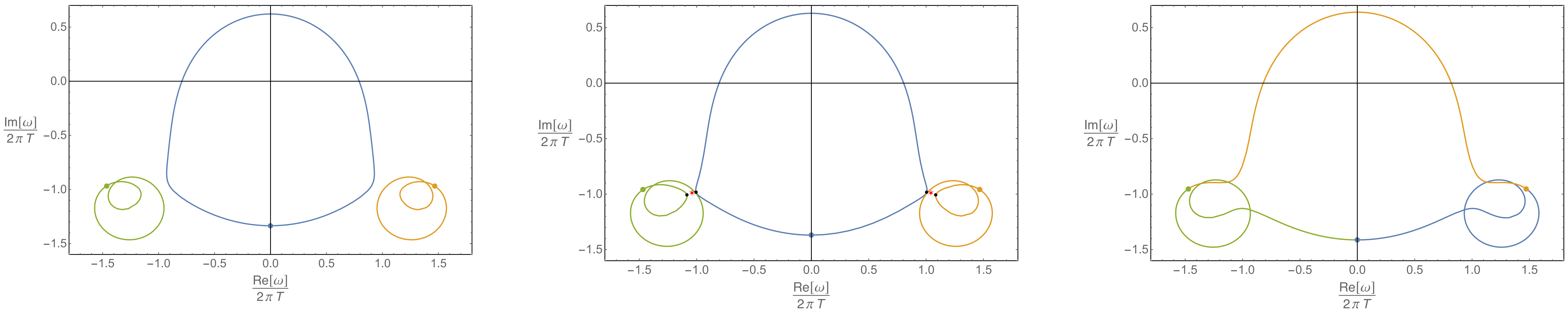
$D_c = \frac{\sigma}{\chi}$	v_{eq}	τ_{eq}	τ_L	v_B
$\sim \frac{m}{T^2}$	~ 1	$\sim \frac{m}{T^2}$	$\frac{1}{2\pi T}$	$\sim \frac{T}{m}$

- diffusion bound from below and above (T>>m)

$$\boxed{v_B^2\tau_L \ll D_c \lesssim \frac{1}{2}v_{eq}^2\tau_{eq}}$$

Results - slow mode

- if we increase temperature, pole collision can occur at complex momentum between the charge diffusive mode and two gapped modes with real parts



Results

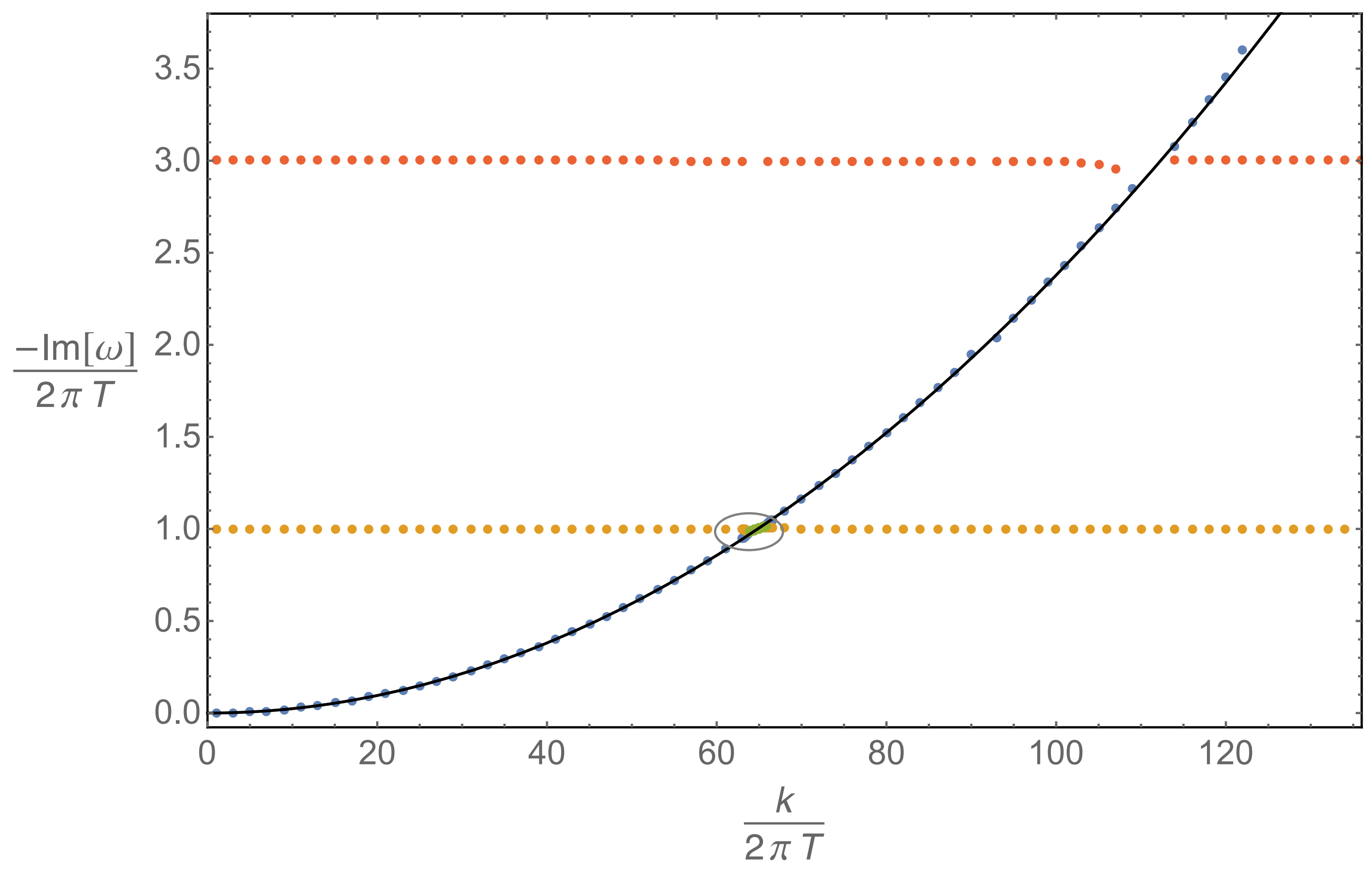
case 2: hydro mode collides with an IR mode

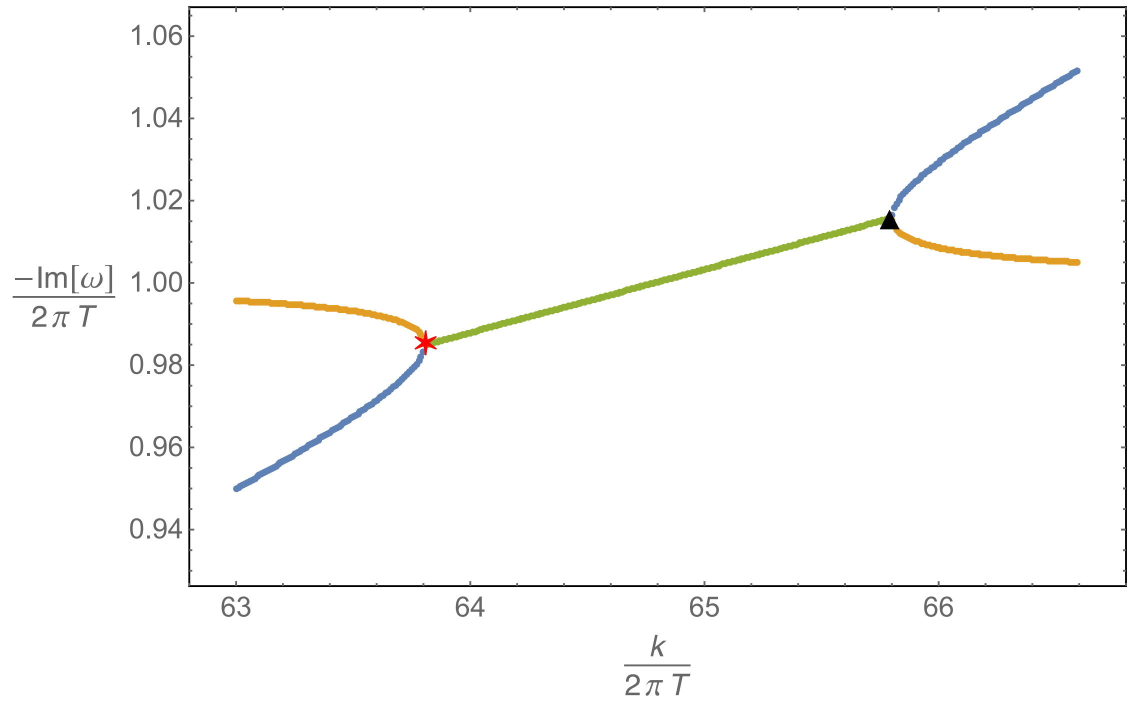
Results - IR mode

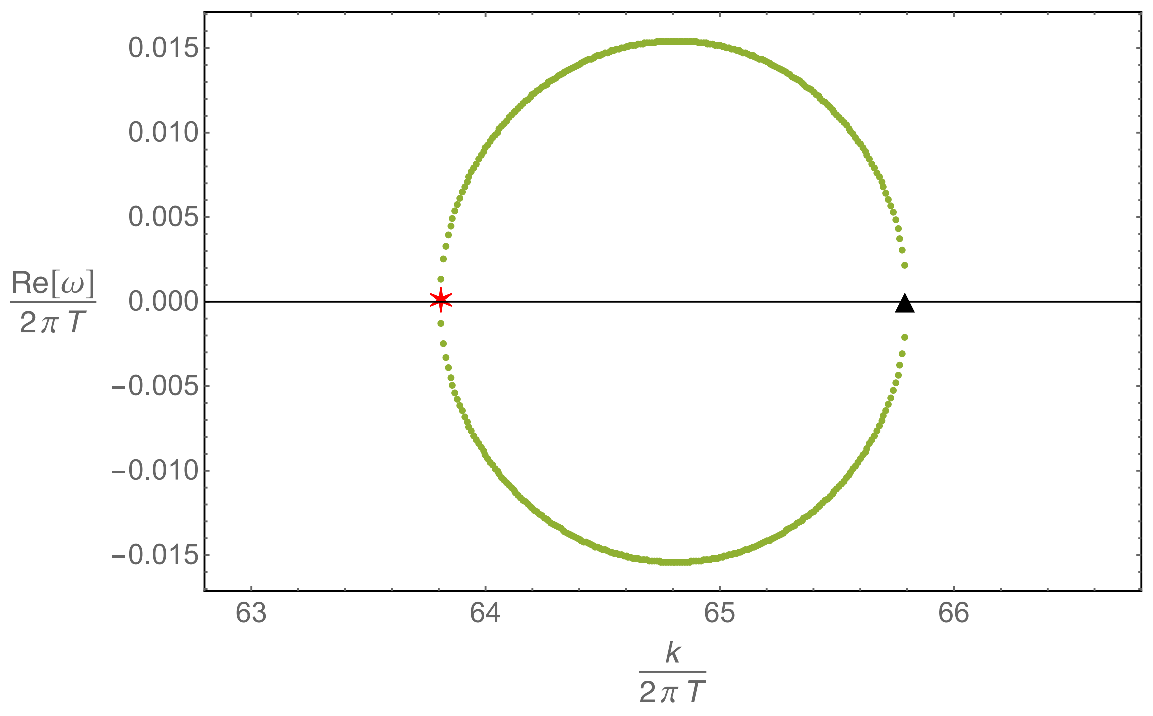
The first non-hydro mode is an IR mode, it collides with charge diffusive mode

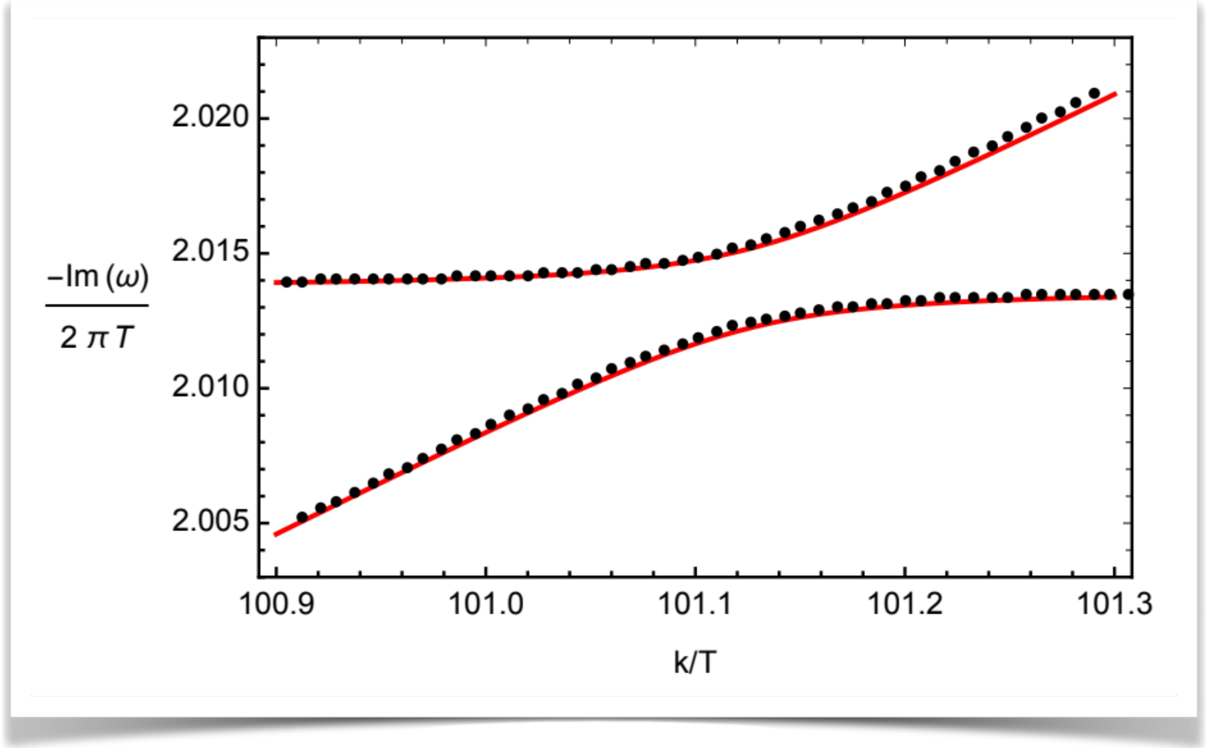
- collision occurs between an *IR* mode and the *diffusive* mode

$$T \to 0$$









charge vs energy, crystal

similarity: collides with 1st IR mode

difference: collision at *real* momentum

$$\boxed{v_B^2\tau_L \ll D_c \lesssim v_{eq}^2\tau_{eq}}$$

similar result can be found in Huh, Jeong, Kim, Sun 2111.07515

Results

hydro mode collides with a slow mode or an IR mode?

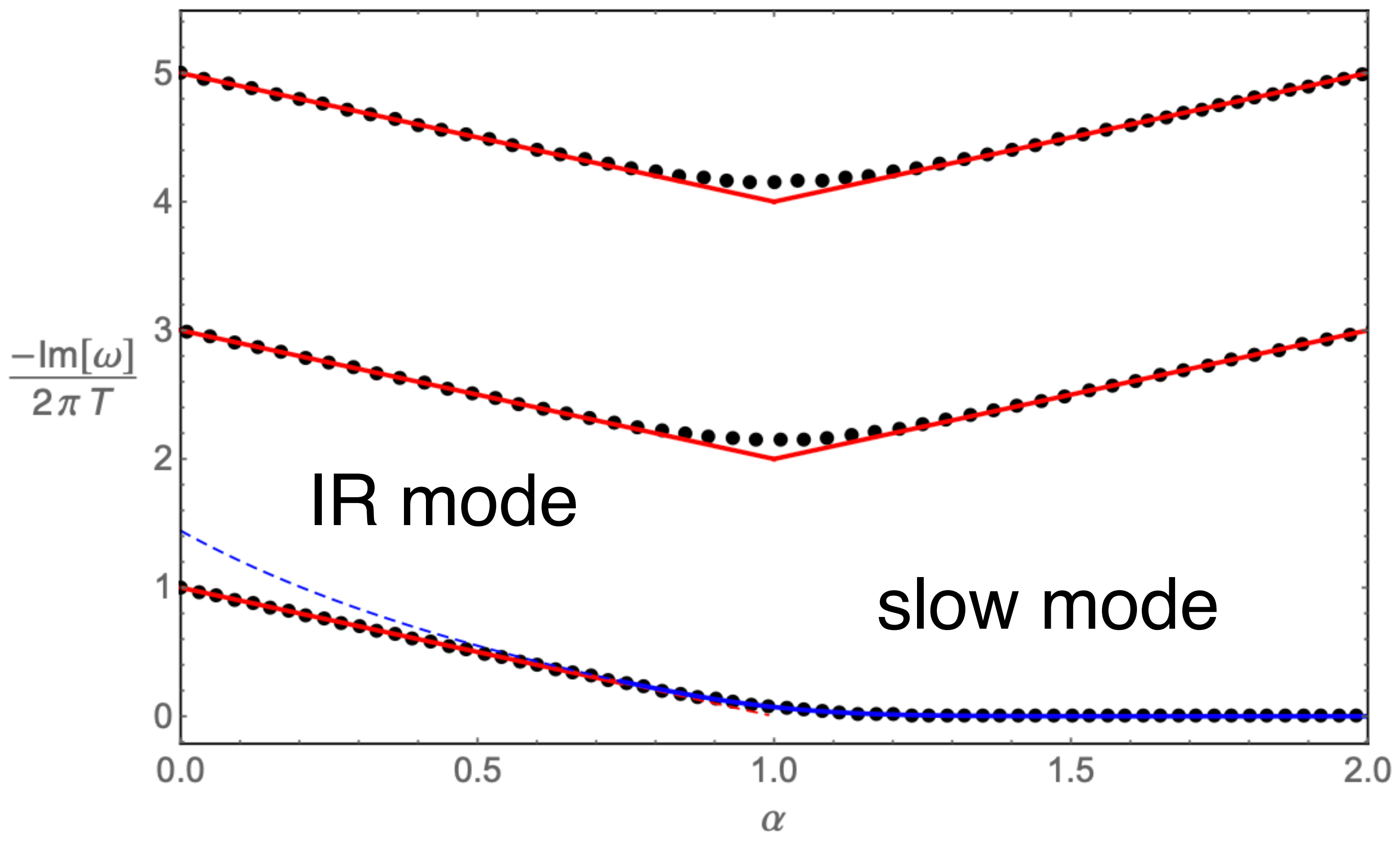
Is there any connections within them?

Yes! $g_{eff}^2 = e^{-\alpha\phi}$ plays an interesting role

Results - general effective gauge coupling

$$g_{eff}^2 = e^{-\alpha\phi}$$

- IR mode to slow mode crossover

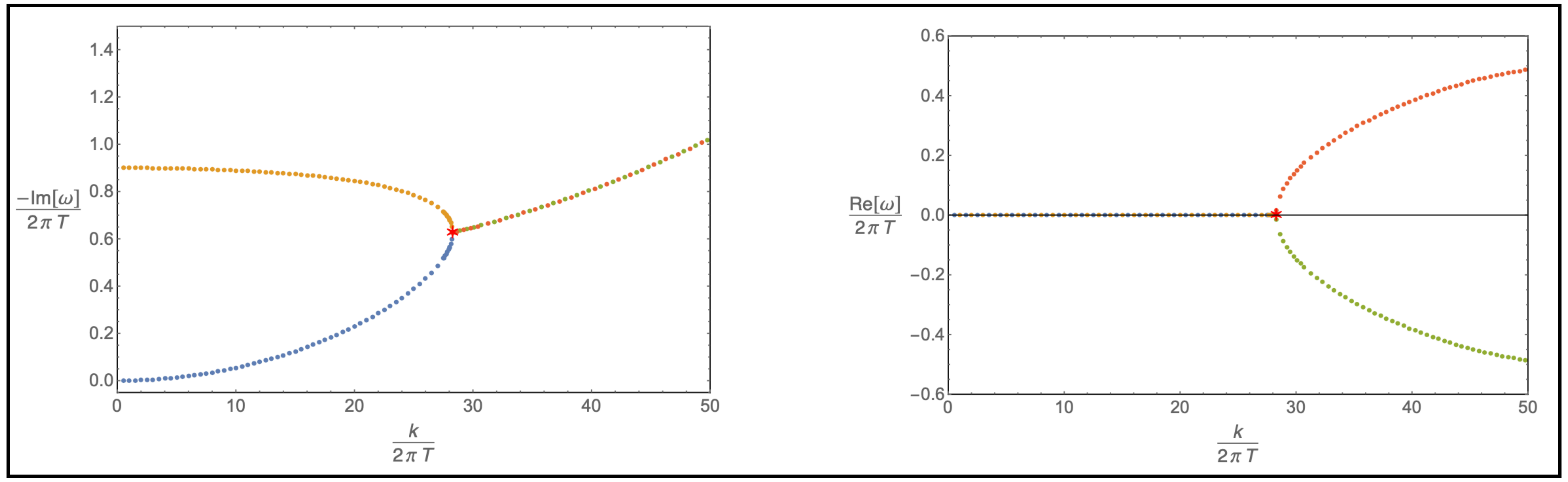


lowest non-hydrodynamic mode	timescale	g_{eff}	effective theory
IR mode	$\tau \sim \frac{1}{T}$	large	?
slow mode	$T\tau \gg 1$	small	quasi-hydrodynamics

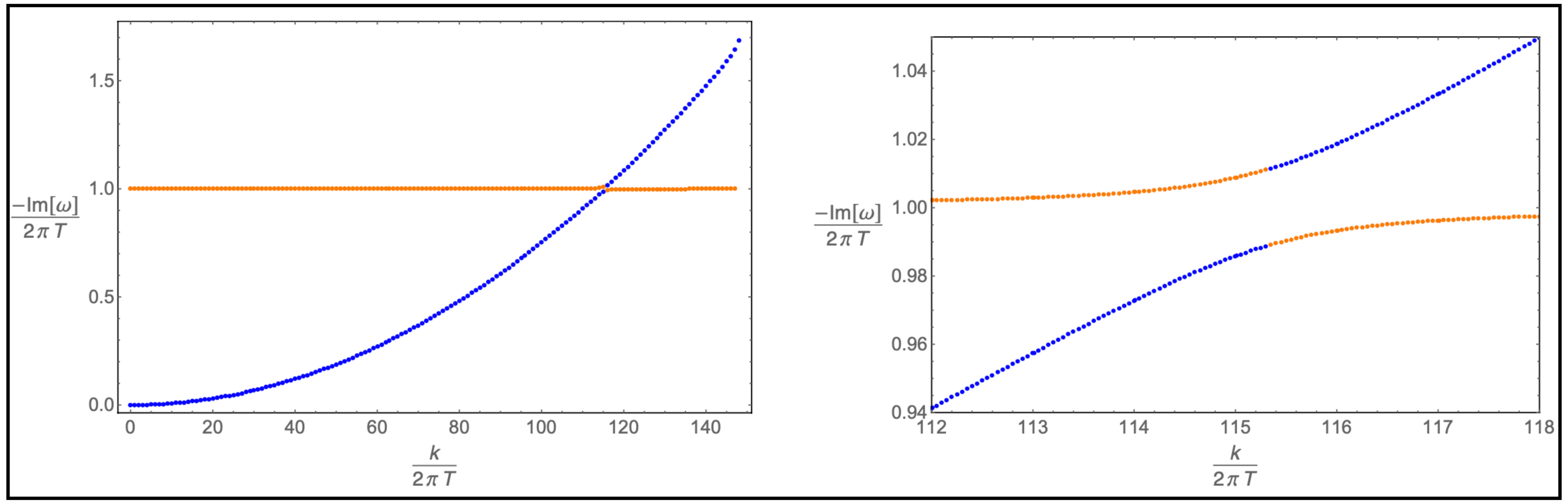
the first non-hydrodynamic mode can be predicted from either IR pole or the matching method

Results - general effective gauge coupling

- pole collision are not always typical



$\alpha = 0.1$



$\alpha = -10^{-4}$

Results - general effective gauge coupling

- convergence of hydrodynamics obey a simple scaling law at low temperature

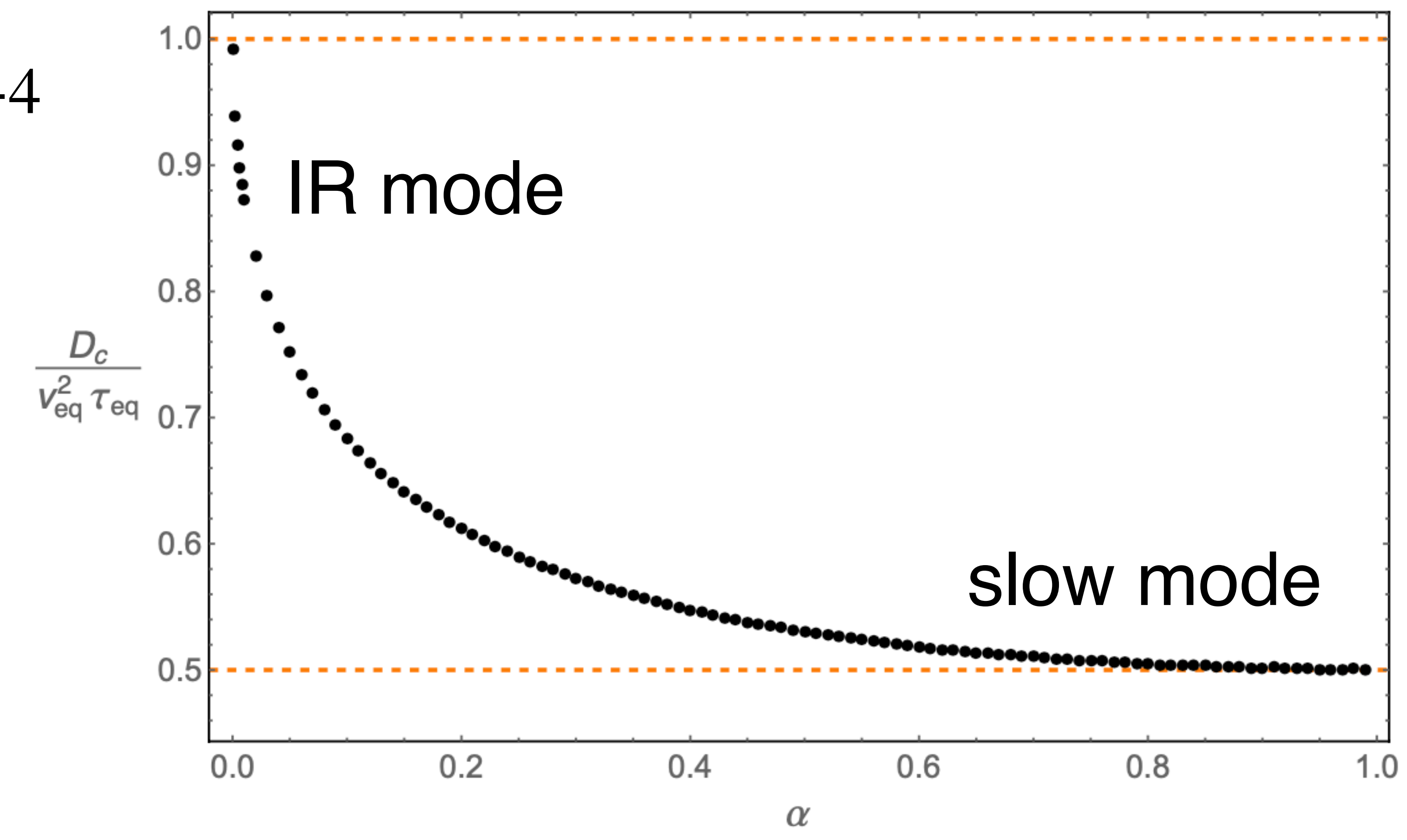
$\alpha \leq 1$ alomst IR mode	$\omega_{eq} \sim T\,, \quad k_{eq} \sim m\left(\frac{T}{m}\right)^{\frac{1+\alpha}{2}}$	$v_{eq} \sim \left(\frac{T}{m}\right)^{\frac{1-\alpha}{2}}\,, \quad \tau_{eq} \sim \frac{1}{T}$
$\alpha > 1$ alomst slow mode	$\omega_{eq} \sim k_{eq} \sim m\left(\frac{T}{m}\right)^{\alpha}$	$v_{eq} \sim T^{0}\,, \quad \tau_{eq} \sim \frac{m^{\alpha-1}}{T^{\alpha}}$

$$D_c \to \frac{2\sqrt{2}}{(1+\alpha)m}\left(\frac{m}{2\sqrt{2}\pi T}\right)^{\alpha}$$

- diffusion upper bound can always be defined with v_{eq}, τ_{eq}

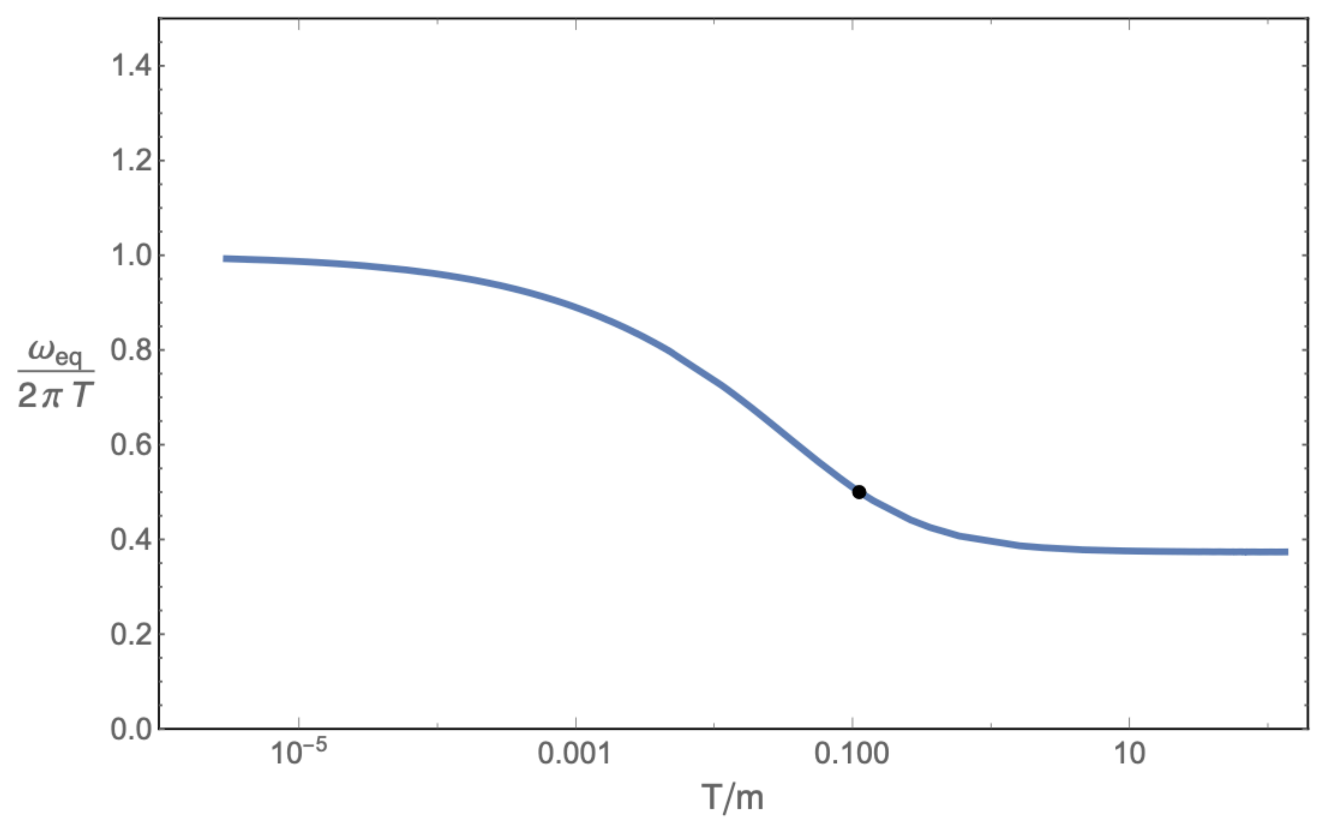
$$D_c \to c(\alpha) v_{eq}^2 \tau_{eq} \quad \text{as} \quad T \to 0$$

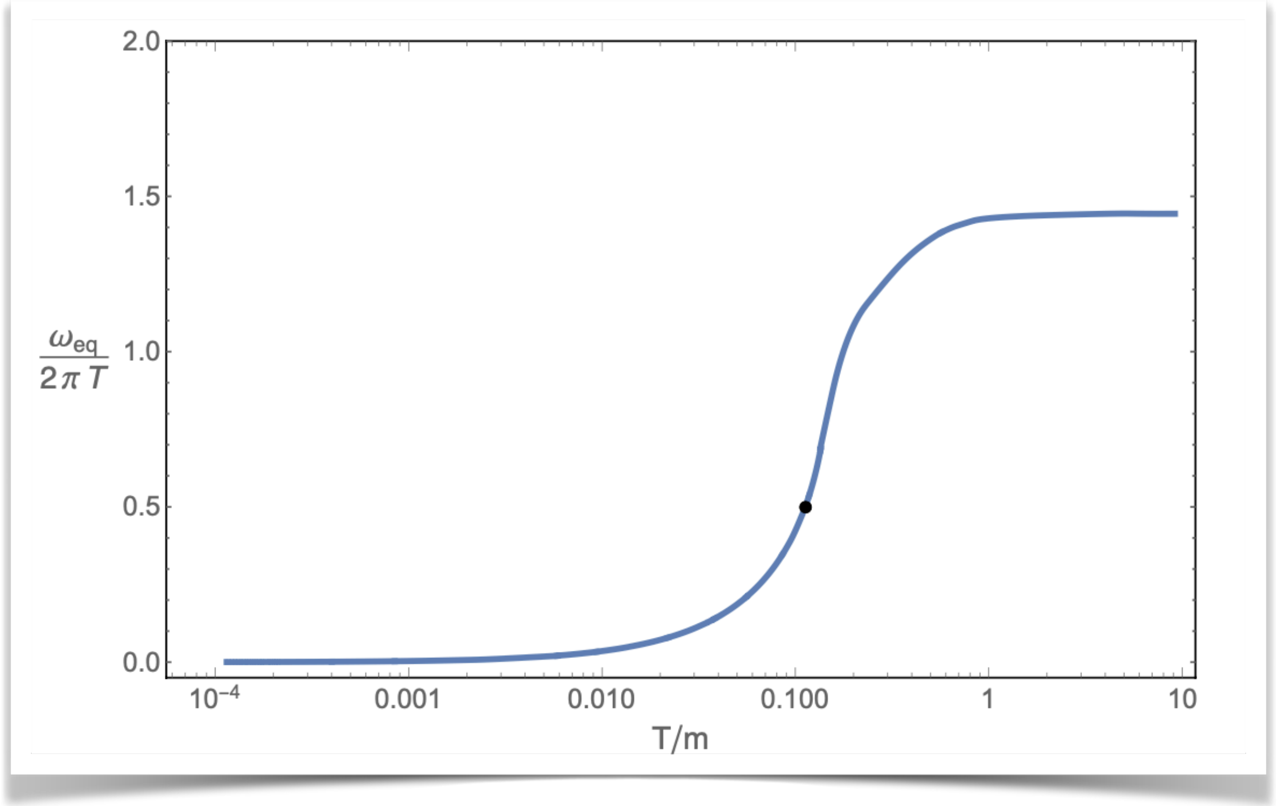
1 for negative $\alpha = 10^{-4}$

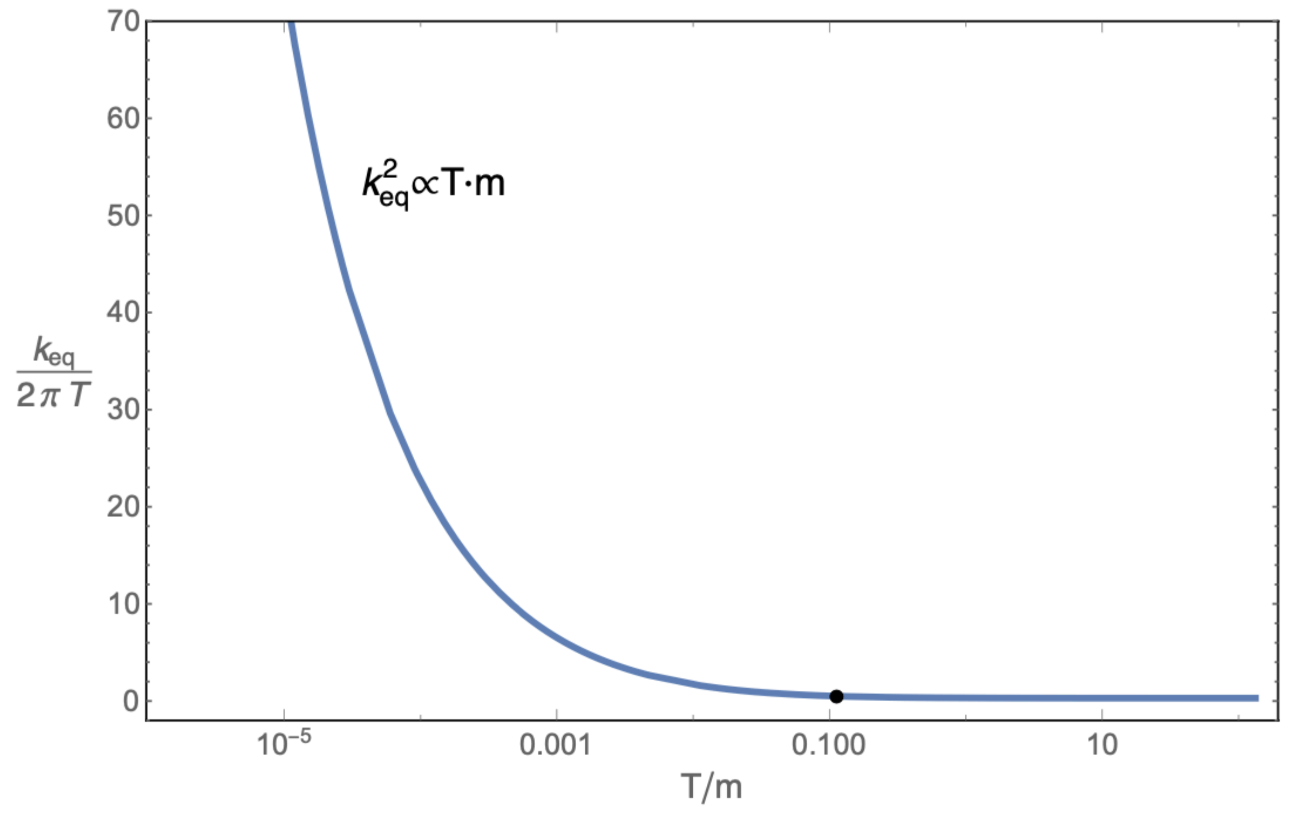


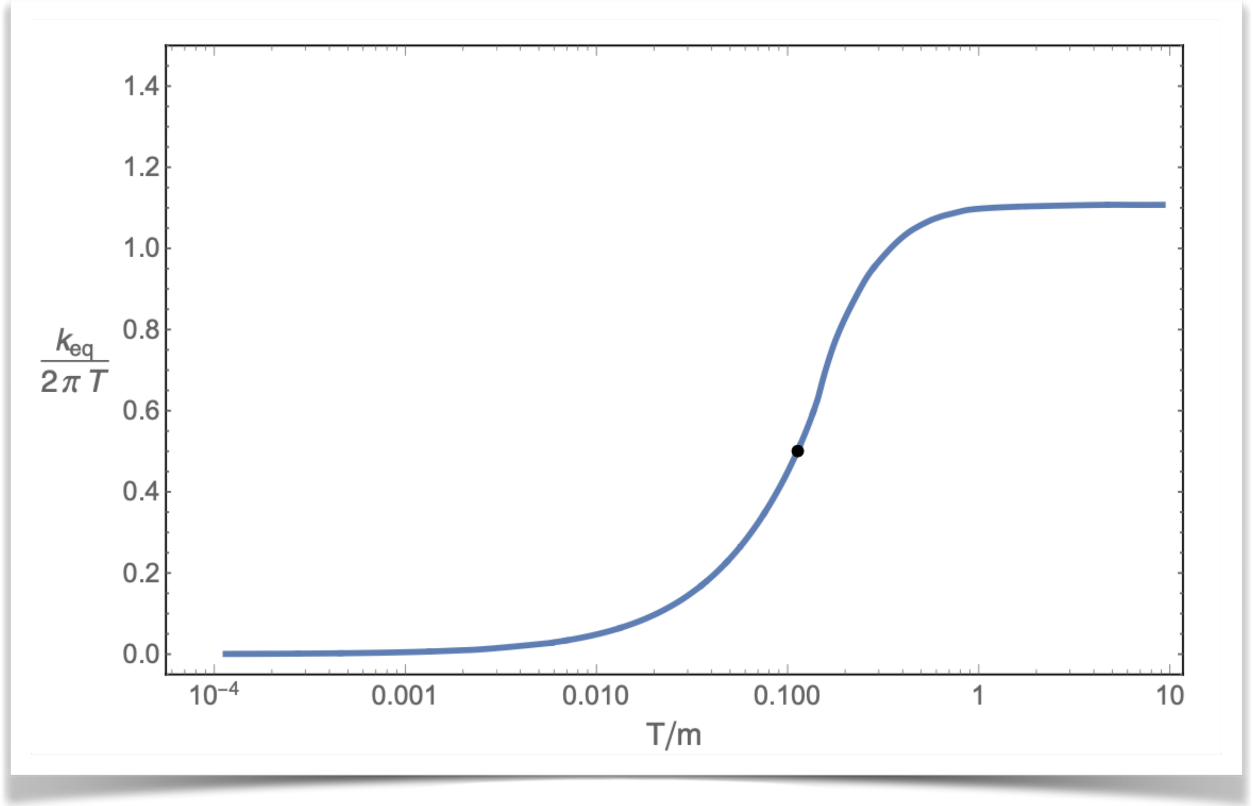
Results - high temperature

- if we increase temperature









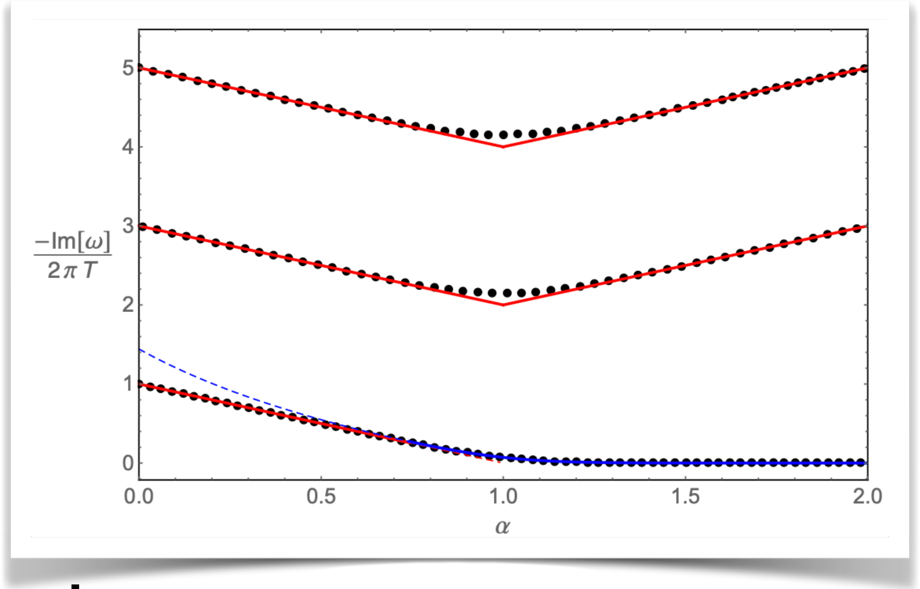
IR mode, $\alpha = 0$

slow mode, $\alpha = 2$

Summary - clues to formulate an EFT

- we considered *charge diffusion* in a *semi-local quantum liquid,* the gauge coupling is tunable $g_{eff}^2 = e^{-\alpha\phi}$

- when the gauge coupling is large, the 1st non-hydrodynamic mode is the IR mode;
  as we decrease this coupling, the 1st non-hydrodynamic mode becomes a slow mode $T\tau \gg 1$



- for IR mode, it seems the 1st excitation is only sensitive to the IR geometry,
  while for slow mode, it depends on the whole geometry, and it is captured by the quasi-hydrodynamics.

- convergence radius of hydrodynamic is obtained from pole collision (next critical point with respect to $(\omega, k) = (0,0)$), and shows scaling behaviors

- charge diffusion constant always have the same scaling dependence with $v_{eq}^2\tau_{eq}$ at $T \to 0$

- at high temperature, we always have $\omega_{eq} \sim k_{eq} \sim T$, and diffusion upper bound still works well.

Further questions & discussion

- For charge diffusion, we find that the first non-hydrodynamic mode that collides with hydrodynamic mode can be transferred from an IR mode to a slow mode, via decreasing the effective gauge coupling.

Could this modification exist for energy diffusion(fluctuations of spin-2)?

a. Maybe we can consider a finite density system where gravitational and gauge fluctuations couple with each other, the equilibration quantities of charge might also influence the equilibration process of energy.

b. Or we can consider a neutral state, but introduce irrelevant deformations.
For charge diffusion in neutral state, the background is fixed. What we can do is to switch how the spin-1 field propagate in this geometry, and one possible way is to introduce an effective gauge coupling.

For energy diffusion, we can consider a set of same IR geometry that represents the same IR fixed point at T=0.
Then we can introduce different irrelevant deformations that flows the IR geometry to AdS boundary.
What we expect is the propagation of the relevant fluctuations might be sensitive to such deformations.

Further questions & discussions

- how to make the corrections to the dispersion relation?

$$\omega = \sum_{n=1}^{\infty} c_n (k^2)^n \qquad c_n = c_n(T, m, \alpha \ldots)$$

- for system with a slow mode, there is a quasi-hydrodynamic picture beyond the hydrodynamic description,

  what about the general cases?

  what is the role of the effective gauge coupling?

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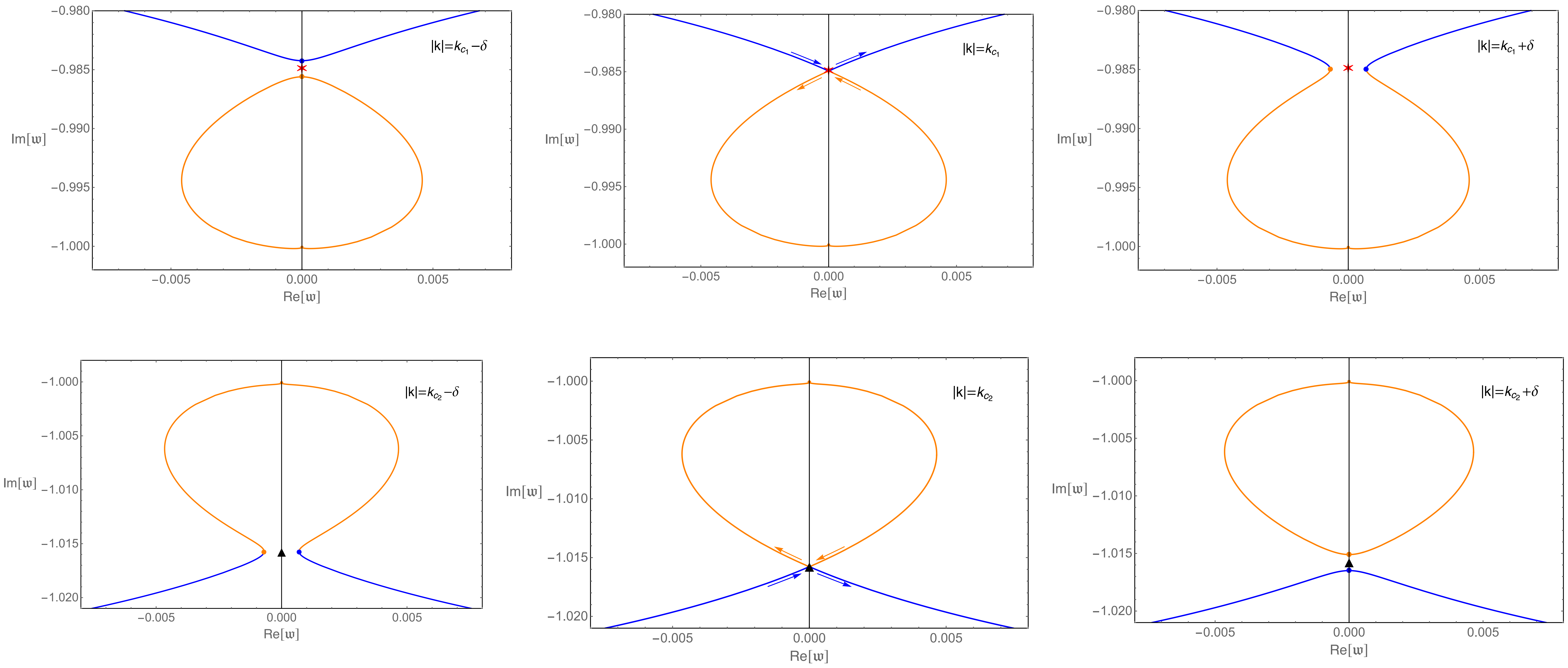
Thanks！

Results - IR mode

The first non-hydro mode is an IR mode, it collides with charge diffusive mode

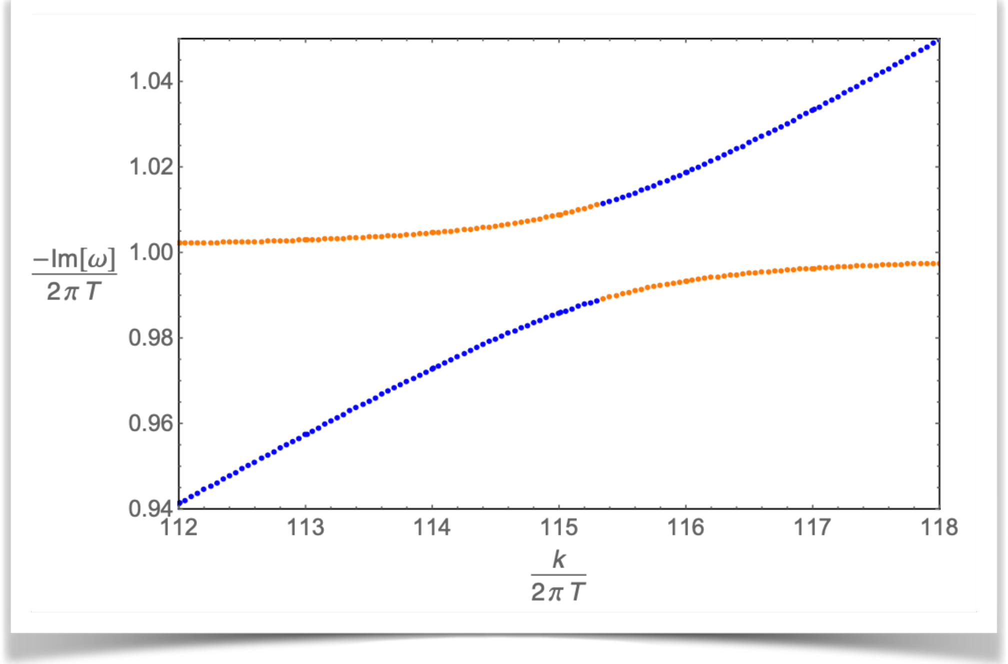
- complex spectral curve shows this is a level crossing

  between IR and diffusive modes, *twice*.



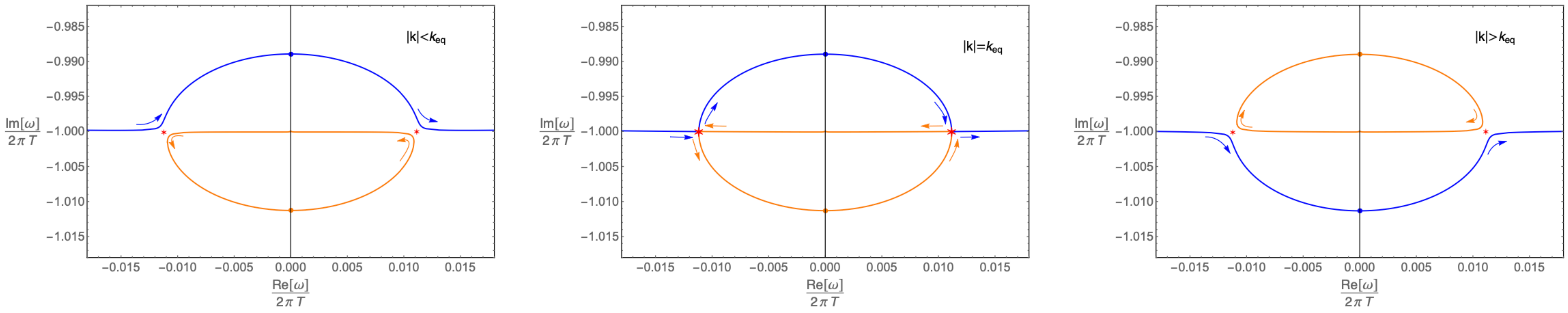
Results - general effective gauge coupling

- and pole collision can even occurs at complex momentum



$$\boxed{k = k_{eq} e^{i\varphi_k}, \omega = \omega_{eq} e^{i\varphi_\omega}}$$

$$\boxed{\varphi_\omega \simeq \varphi_k - \pi/2}$$



this is similar to the cases for avoided pole collision in energy, crystal diffusions