2022.01.17 @ SJTU

Dynamical stability and filamentary instability in holographic conductors

Masataka Matsumoto (Shanghai University; 上海大学)

S. Ishigaki, S. Kinoshita, and MM, arXiv:2112.11677[hep-th]

Filamentary instability

Nonlinear conductivity

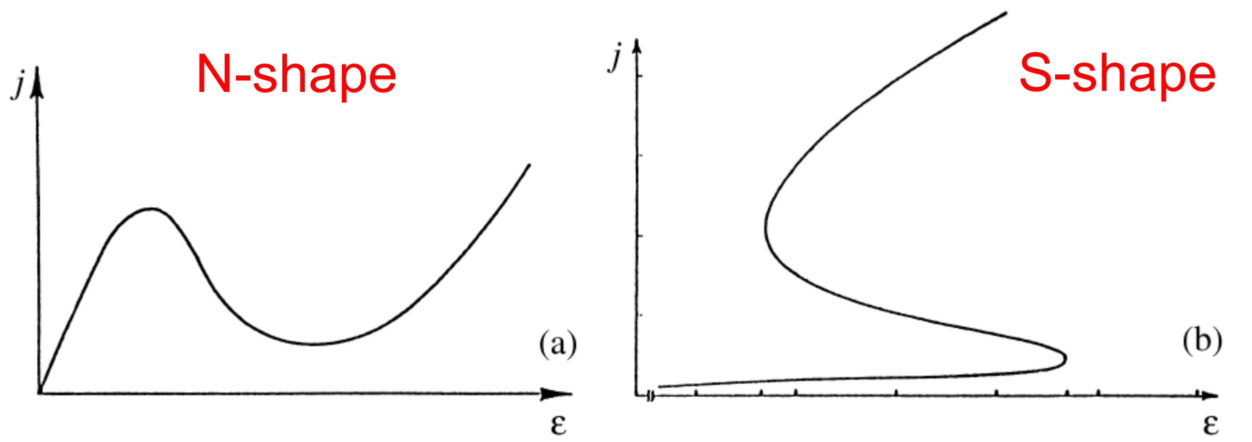
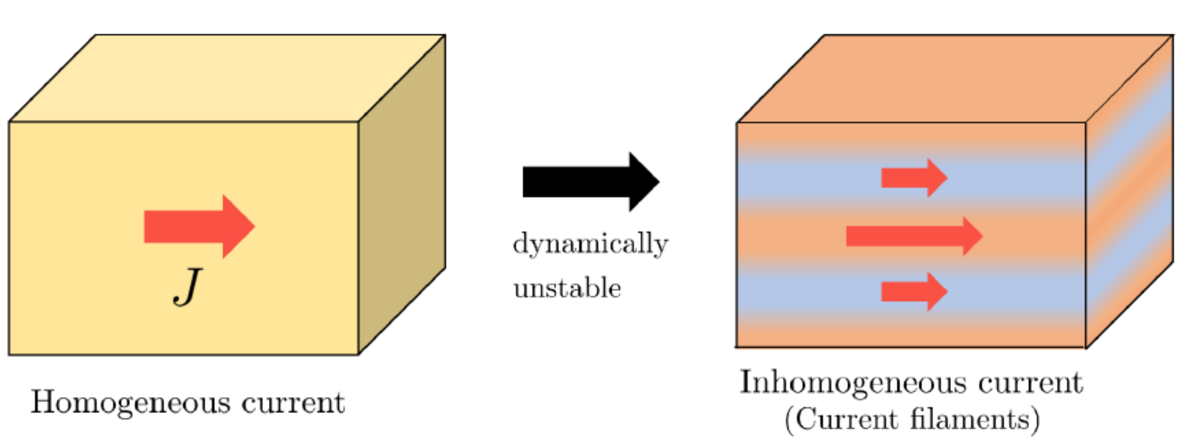


Figure 1.1. The current density j versus the electric field $\mathcal{E}$ for two types of negative differential conductivity (NDC): (a) NNDC and (b) SNDC (schematic).

In the S-shape J-E characteristics, the current is a multivalued function of the electric field.

E. Scholl, Nonlinear Spatio-Temporal Dynamics and Chaos in Semiconductors, Cambridge, University Press (2001).

In the SNDC case, current filamentation generally occurs (filamentary instability).



The S-shape J-E characteristics can be realized in holography.
Does it show the filamentary instability?

Probe brane model: D3/D7 model

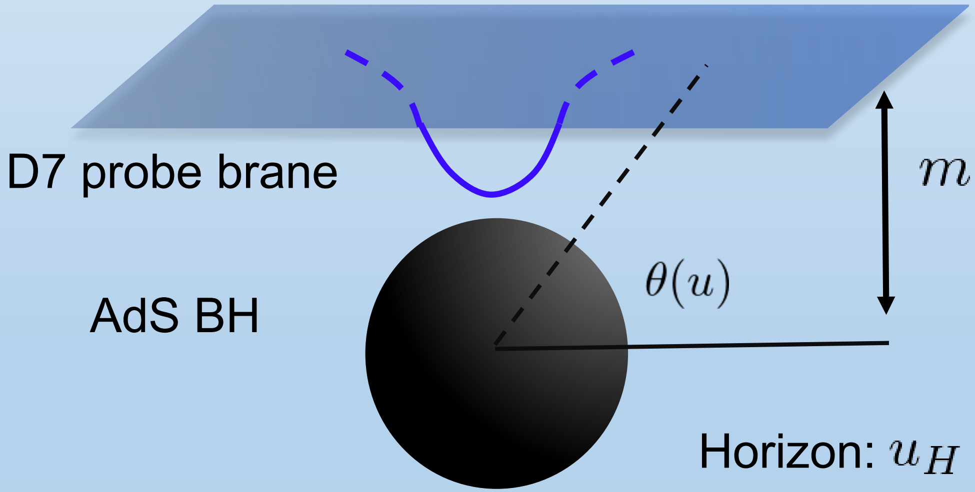
AdS-BH background
$$ds^2 = \frac{1}{u^2}\left(-f(u)dt^2 + d\vec{x}^2 + \frac{du^2}{f(u)}\right) + d\Omega_5^2, \qquad f(u) = 1 - \frac{u^4}{u_H^4}$$

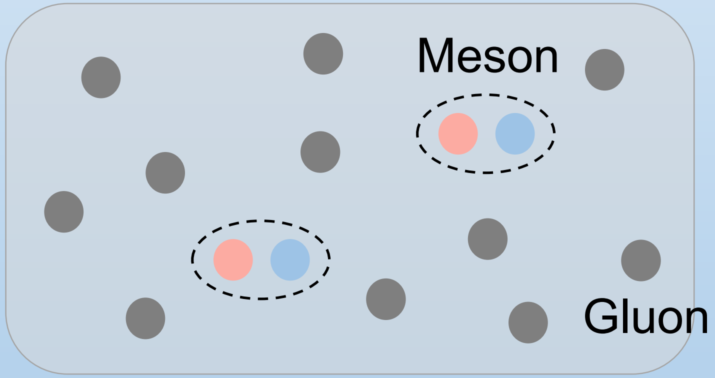
The D7 probe brane action
$$S_{D7} = -T_{D7}\int d^8\xi\sqrt{-\det\left(g_{ab} + (2\pi\alpha')F_{ab}\right)} + \frac{(2\pi\alpha')^2}{2}T_{D7}\int C^{(4)}\wedge F\wedge F$$

AdS black hole + D7 probe brane

(3+1) dim. N=4 SYM + N=2 quark multiplet

Karch, Katz (2003).





The embedding function:
$$\theta(u) = mu + \theta_2 u^3 + \cdots$$

Heat bath temperature: $T = \dfrac{1}{\pi u_H}$

Quark mass: m

Chiral condensate: $c \sim \left(2\theta_2 - \dfrac{m^3}{6}\right)$

Holographic conductors

Switch on the gauge fields: $A_x(t,u) = -Et + h(u)$

$$\sim -Et + \frac{J}{2}u^2 + \cdots\,,$$

E : external electric field

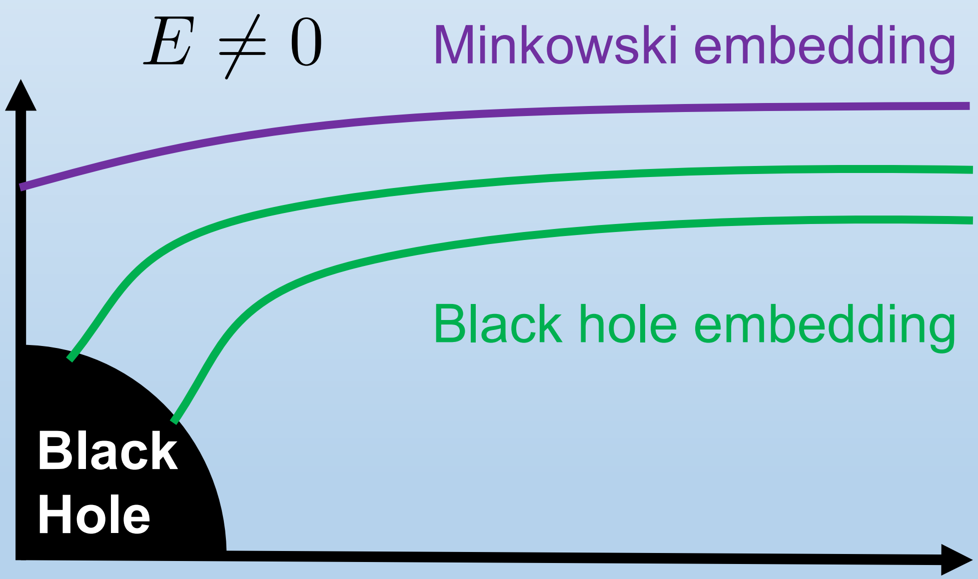
J : current density

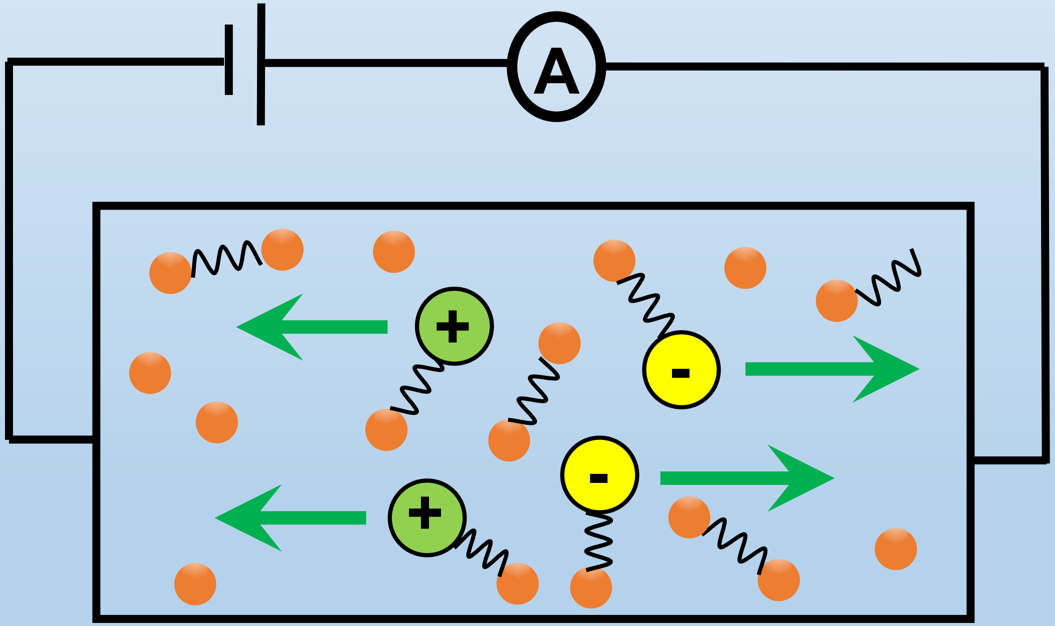
Karch, O'Bannon (2007).

$\boxed{J = 0 \text{ : Insulating phase}}$

$E \neq 0$ or $\boxed{J \neq 0 \text{ : Conducting phase}}$

<u>Two different phases: insulating or conducting</u>





Dimensionless parameters: $\left(\frac{T}{m}, \frac{E}{m^2}\right)$

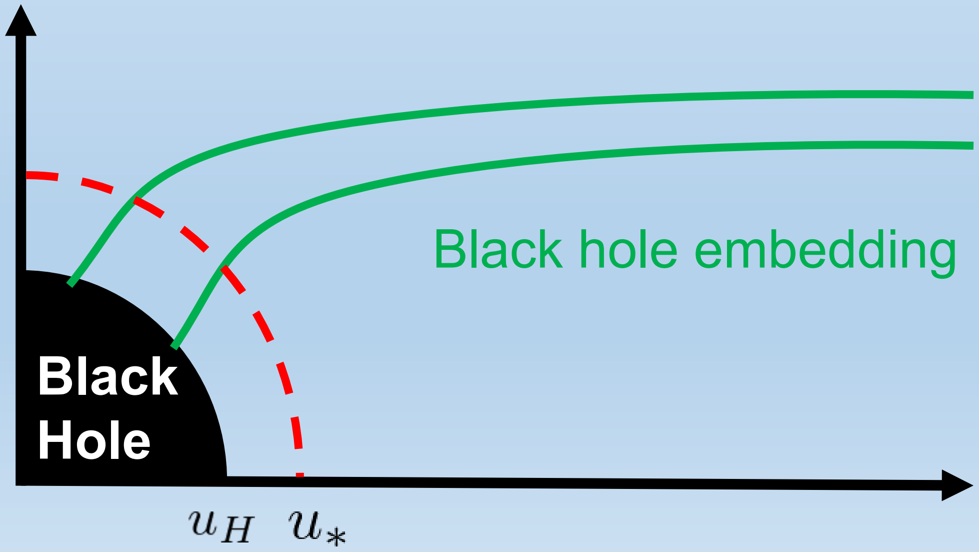
In the Black hole embedding, the dual system has a constant current flow.
The system is in non-equilibrium steady state (NESS).

Holographic conductors

In conducting phase (Black hole embedding), the effective horizon appears.
The effective horizon is a causal boundary with respect to the fluctuations on the probe brane.

$$\gamma_{ab}\mathrm{d}\xi^a\mathrm{d}\xi^b = \frac{g_{tt}g_{xx}+E^2}{g_{xx}}\mathrm{d}t^2 - 2\frac{Eh'}{g_{xx}}\mathrm{d}t\mathrm{d}u + \frac{g_{xx}g_{uu}+h'^2}{g_{xx}}\mathrm{d}u^2 + \left(g_{xx}+\frac{E^2}{g_{tt}}+\frac{h'^2}{g_{uu}}\right)\mathrm{d}x^2 + \frac{\mathrm{d}y^2+\mathrm{d}z^2}{u^2} + \cos^2\theta(u)\mathrm{d}\Omega_3^2.$$

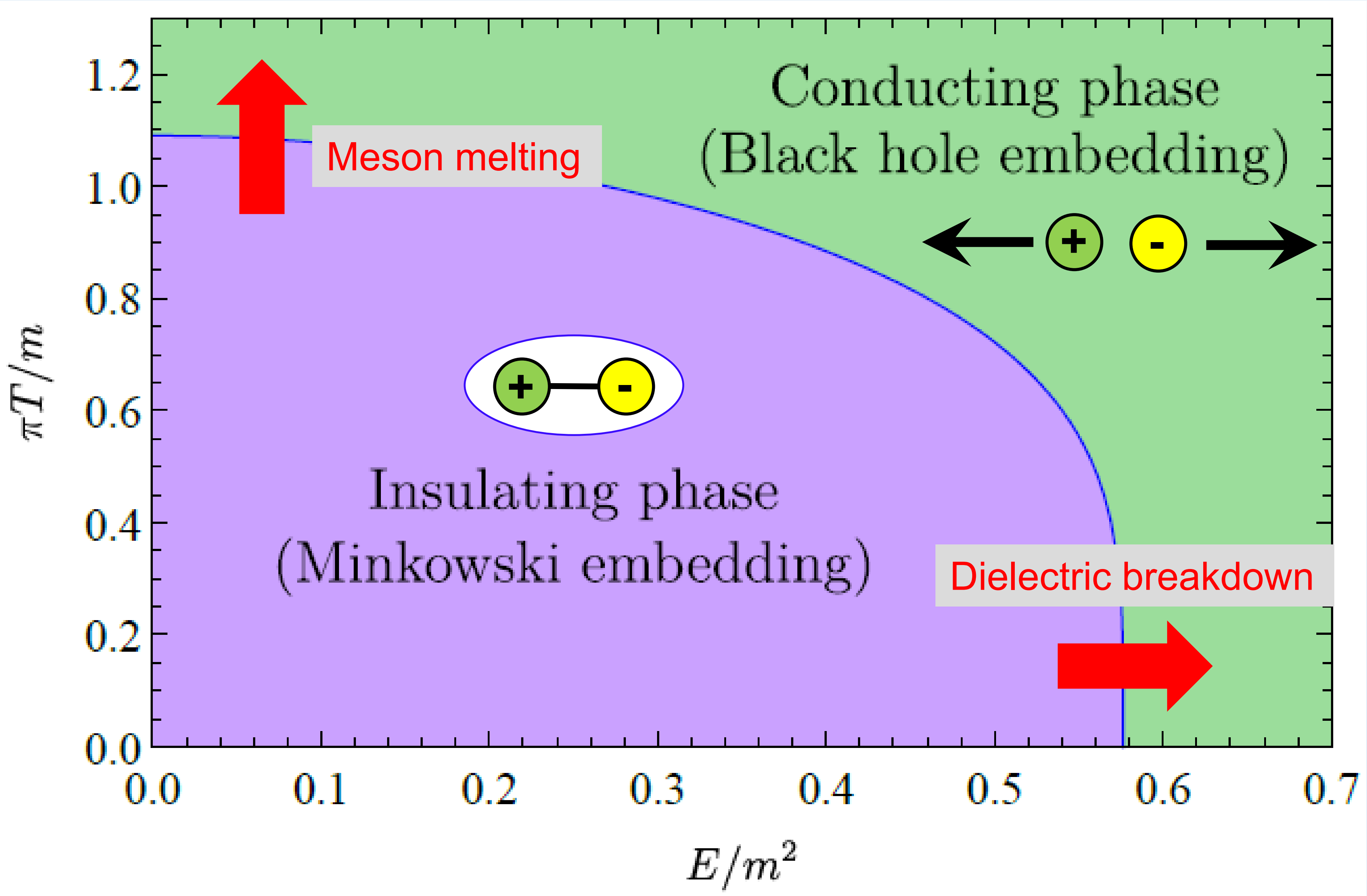
$$\gamma_{ab} \equiv g_{ab} + (2\pi\alpha')^2 F_{ac}F_{bd}g^{cd}$$ *: Open string metric*



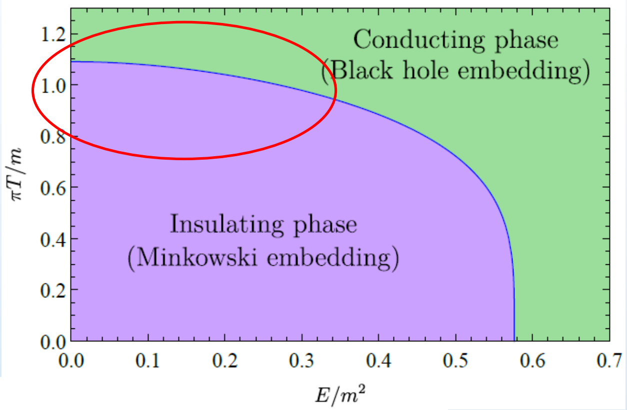
The current density J can be determined locally at the effective horizon.

$$J = \sqrt{-g_{tt}g_{xx}}\cos^3\theta\Big|_{u_*}$$

Phase diagram



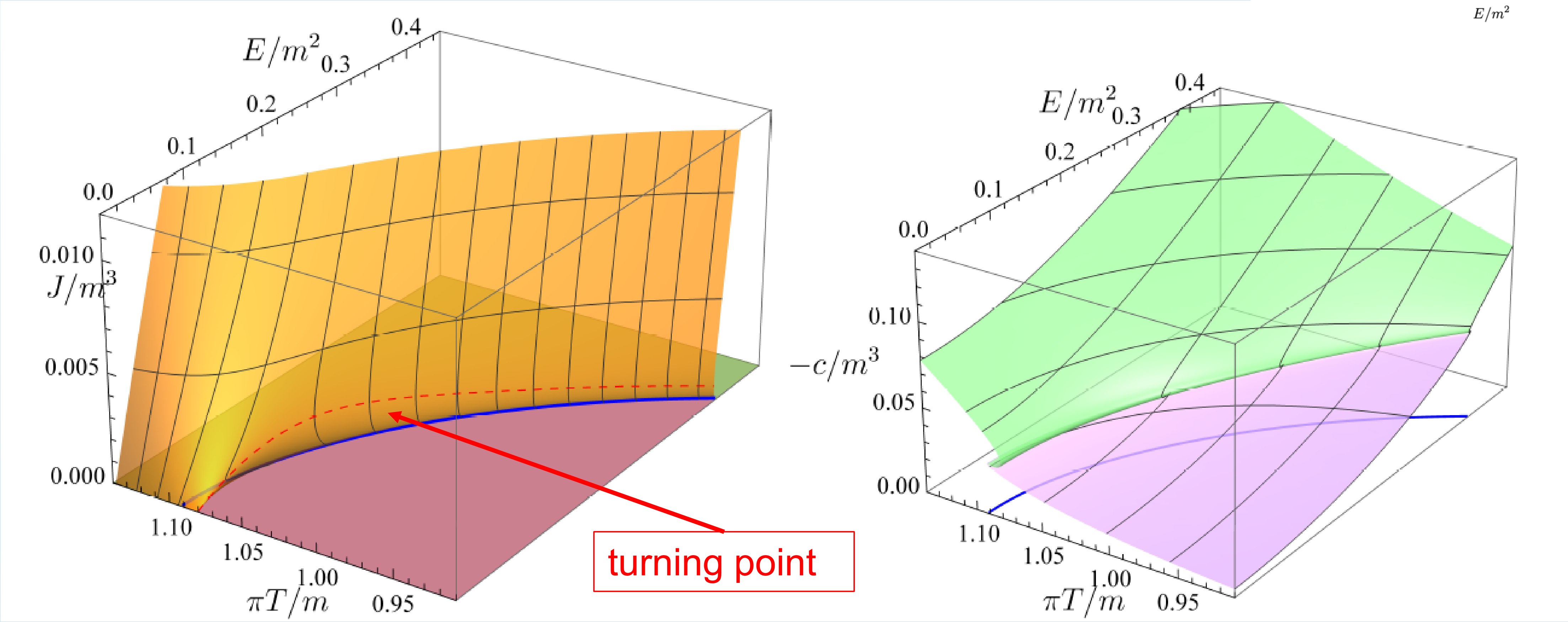
Current density and chiral condensate



Asymptotic form near the AdS boundary

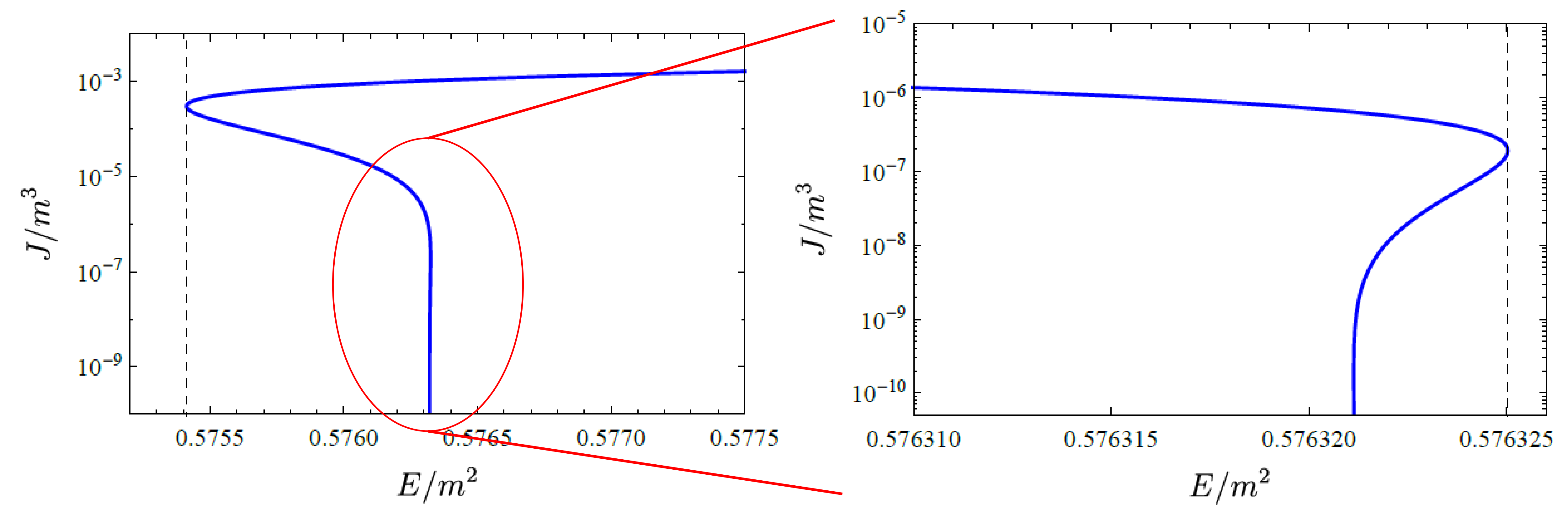
$$\theta(u) = mu + \theta_2 u^3 + \cdots \qquad c \sim \left(2\theta_2 - \frac{m^3}{6}\right)$$

$$A_x(t,u) = -Et + \frac{J}{2}u^2 + \cdots$$



Multi-valued functions near the critical embedding.

Multivaluedness of J-E characteristics at T=0



J is not a two-valued function of E, and expected to be a <u>multi-valued</u> function.

Effective temperature in NESS

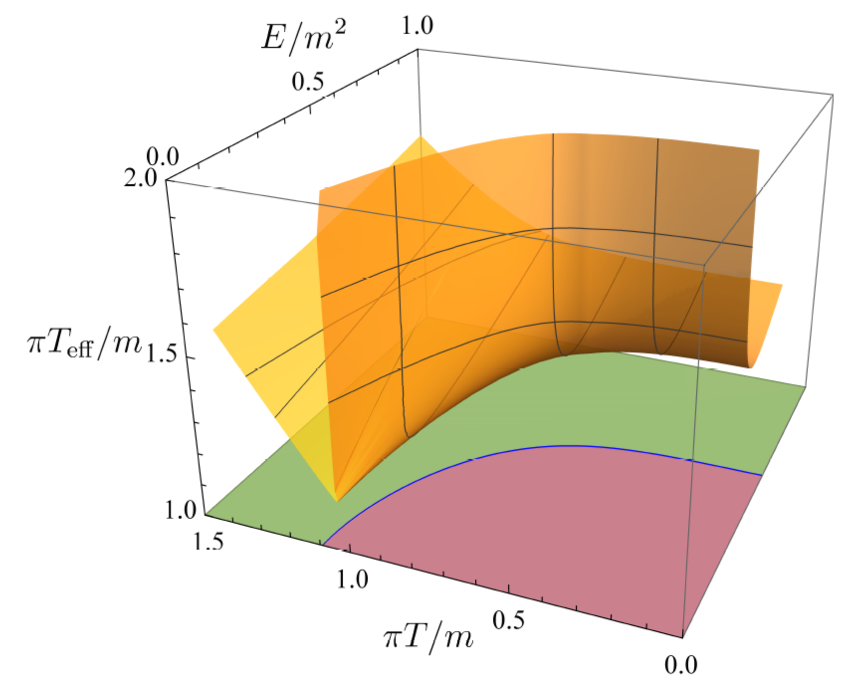
In this system, we can define another temperature from the effective metric.
The effective metric on the probe brane is given by

$$\gamma_{ab}\mathrm{d}\xi^a\mathrm{d}\xi^b = \frac{g_{tt}g_{xx}+E^2}{g_{xx}}\mathrm{d}t^2 - 2\frac{Eh'}{g_{xx}}\mathrm{d}t\mathrm{d}u + \frac{g_{xx}g_{uu}+h'^2}{g_{xx}}\mathrm{d}u^2 + \left(g_{xx}+\frac{E^2}{g_{tt}}+\frac{h'^2}{g_{uu}}\right)\mathrm{d}x^2 + \frac{\mathrm{d}y^2+\mathrm{d}z^2}{u^2} + \cos^2\theta(u)\mathrm{d}\Omega_3^2.$$

$$\gamma_{ab} \equiv g_{ab} + (2\pi\alpha')^2 F_{ac}F_{bd}g^{cd}$$ *: Open string metric*

The effective temperature can be defined by

$$2\pi T_{\mathrm{eff}} = \kappa = \sqrt{\frac{-\gamma_{tt}}{\gamma_{tu}^2 - \gamma_{tt}\gamma_{uu}}}\frac{d}{du}\sqrt{-\gamma_{tt}}\Bigg|_{u=u_*} = -\frac{\gamma'_{tt}(u_*)}{2\gamma_{tu}(u_*)}$$



Perturbations

Consider small perturbations on the background solutions.

$$\theta \to \theta(u) + \delta\theta(u) e^{-i\omega t + ik_\perp x_\perp}$$

$$A_x \to -Et + h(u) + \delta A_x(u) e^{-i\omega t + ik_\perp x_\perp}$$

($x_\perp$ is perpendicular to E)

We obtain the coupled linear-order differential equations with ω and $k_\perp$ for perturbations.
What we want to do is to solve this eigenvalue equations under the following boundary conditions.

At the effective horizon, we impose the ingoing-wave boundary condition.

$$\Phi(u) = \left(1 - \frac{u}{u_*}\right)^{i\lambda} \Phi_{\mathrm{reg}}(u)$$

$$\Phi = (\delta\theta, \delta A_x)^T$$

$\lambda = 0$ corresponds to the ingoing-wave boundary condition.

Details are in Appendix.

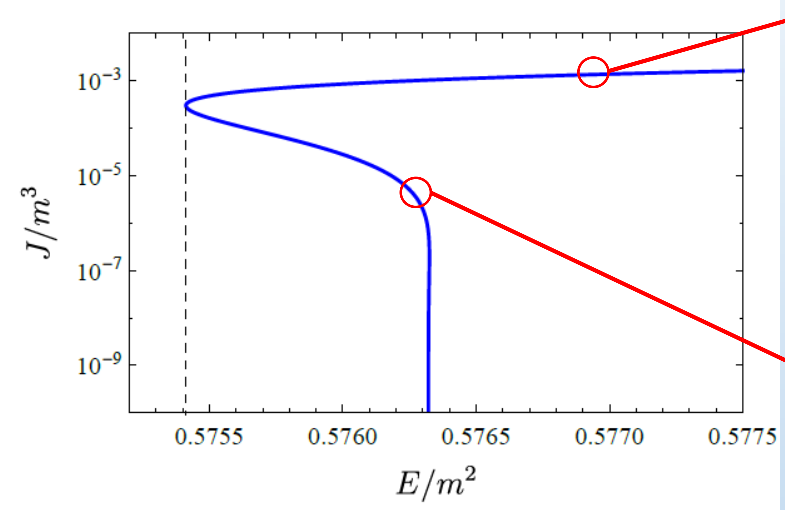
At the bulk boundary, we impose source-free condition.

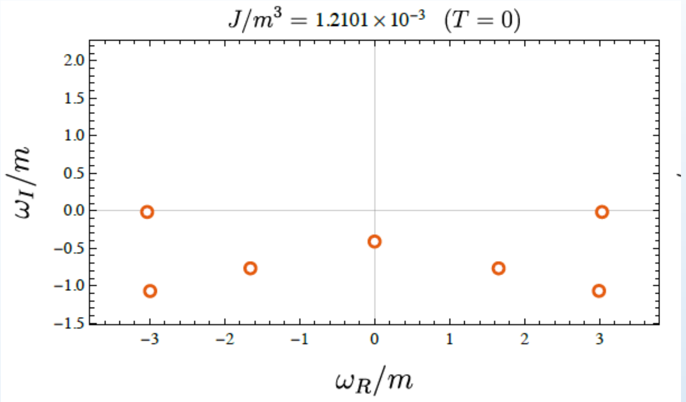
$$\Phi_{\mathrm{reg}}(u = 0) = 0$$

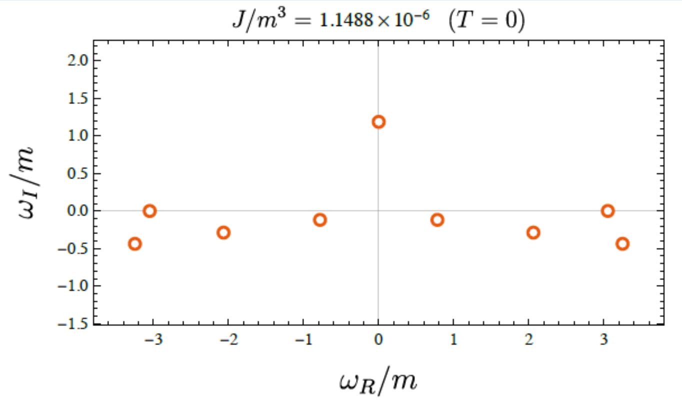
Calculate the eigenvalues ω.
Quasi-normal modes (QNMs)

QNM at T=0

J-E characteristics

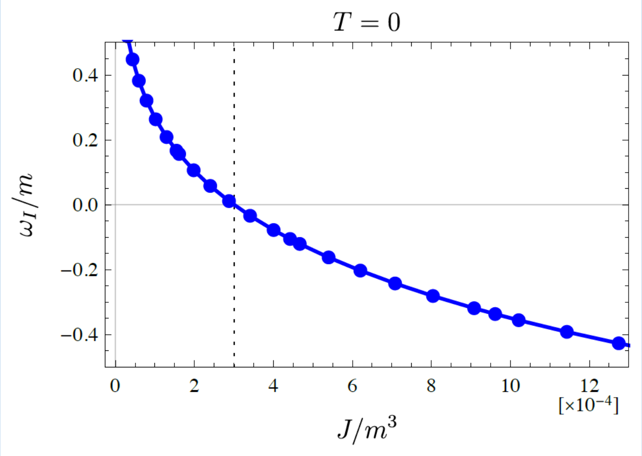




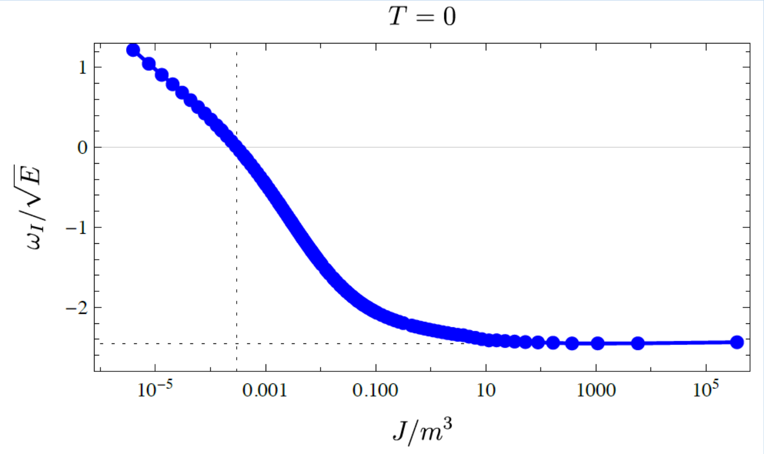


A pure-imaginary mode moves from lower-plane to upper-plane in complex-ω plane.
➡ Perturbations are not damped but growing. (unstable)

This pure-imaginary mode go across the origin at the turning point in J-E characteristics.



for large J
(massless limit)

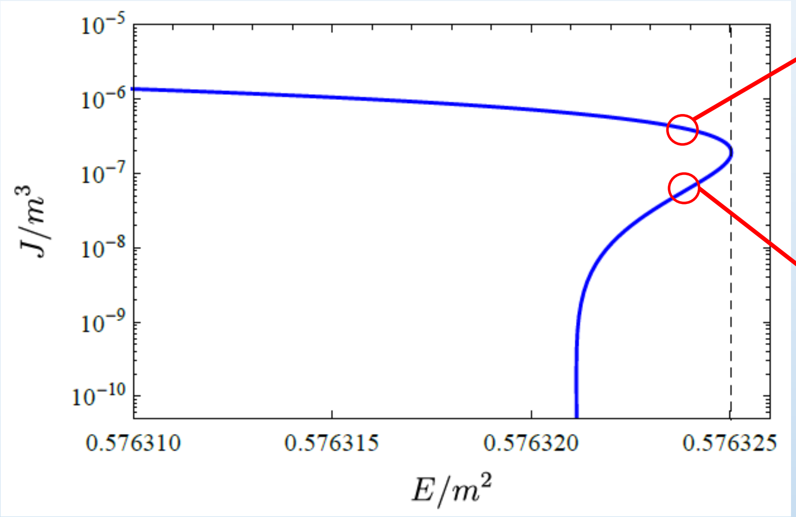


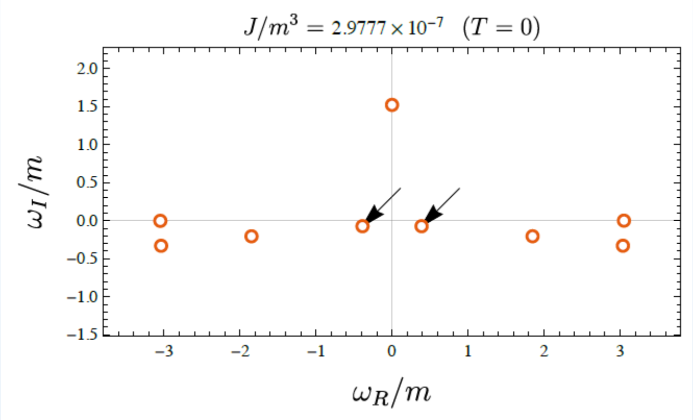
It converges to $-\sqrt{6}$ in massless limit.

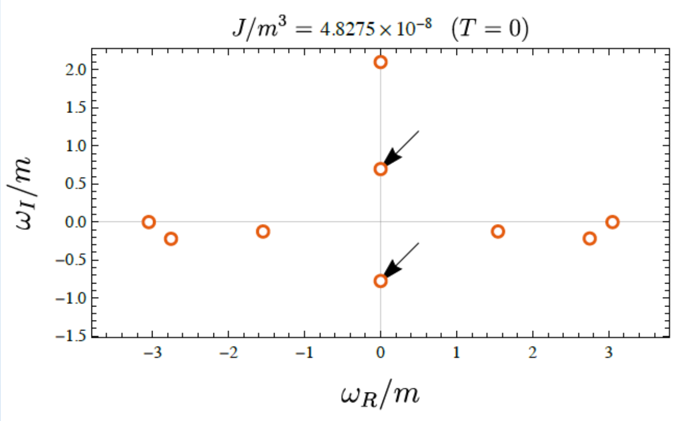
Albash, Filev, Johnson, Kundu (2008).

QNM at T=0

J-E characteristics for small J

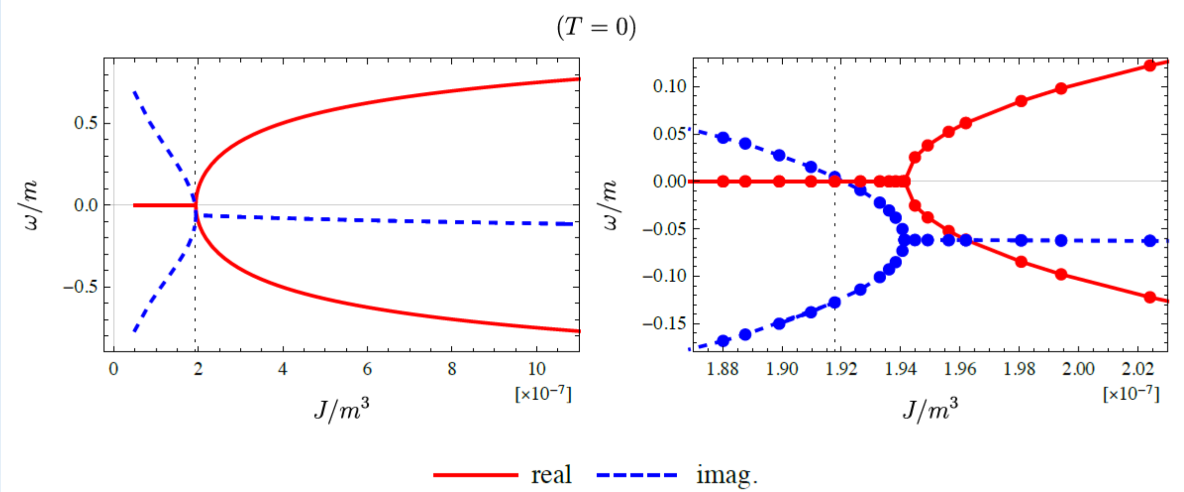






Two modes collide with each other and become pure-imaginary modes.
One of them go across zero again at the second turning point.

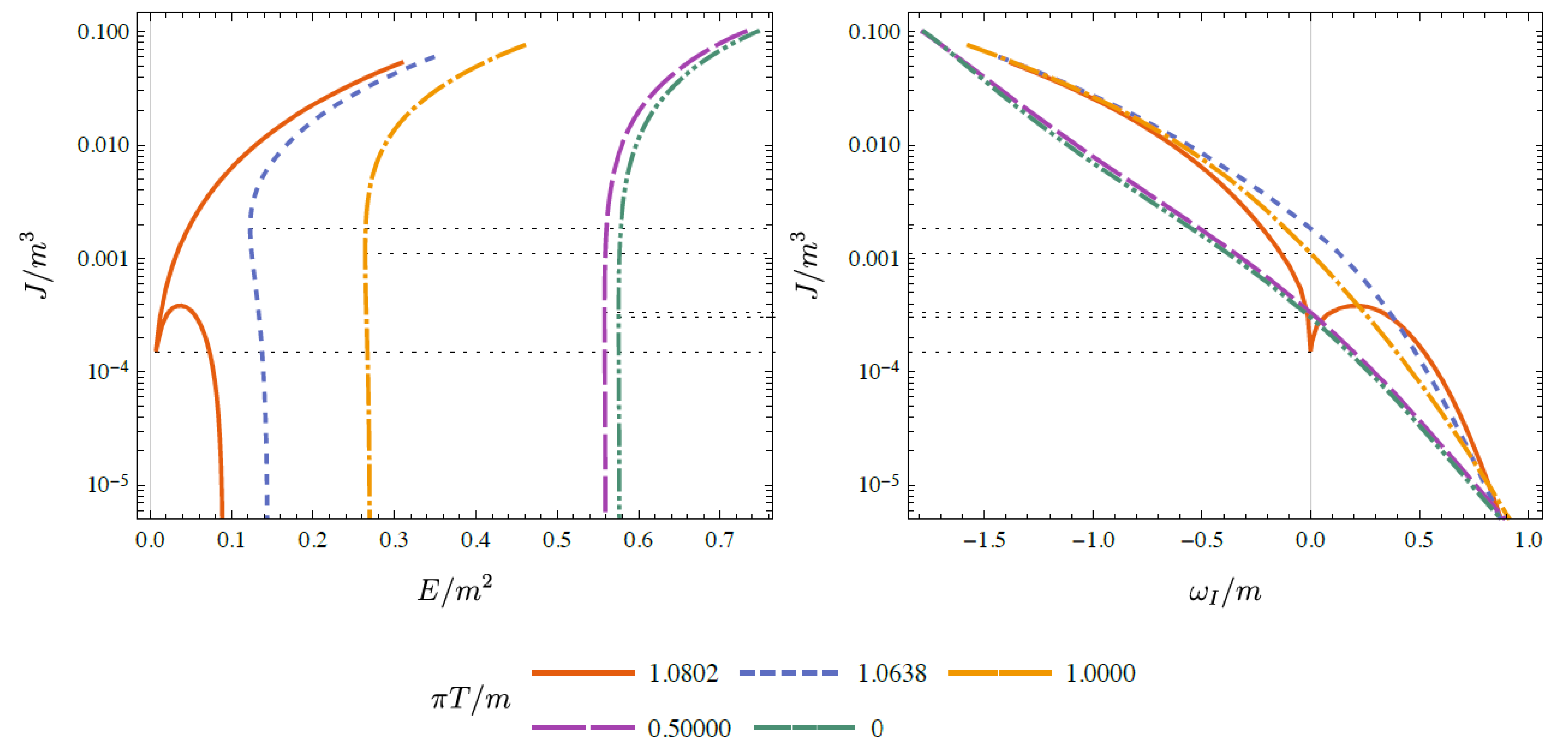
Two QNMs behaviors.



We expect that there are many “turns” in J-E characteristics for small J.

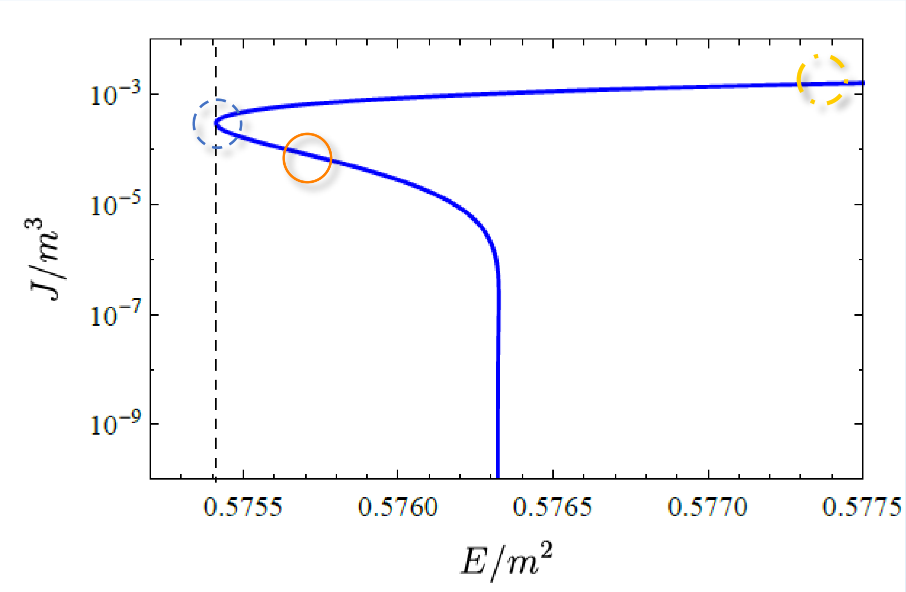
➡ Pure-imaginary modes go across zero at every “turning points”?

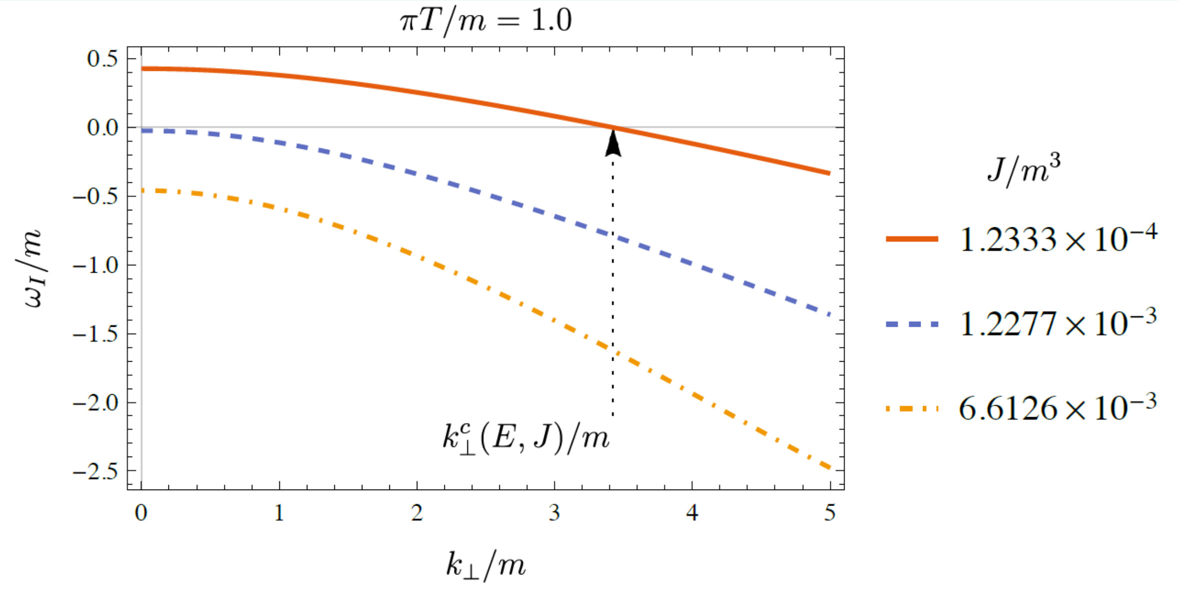
QNM at finite temperature



As in the case of zero temperature, pure-imaginary modes go across zero at the turning points for finite temperature.

Inhomogeneous perturbations





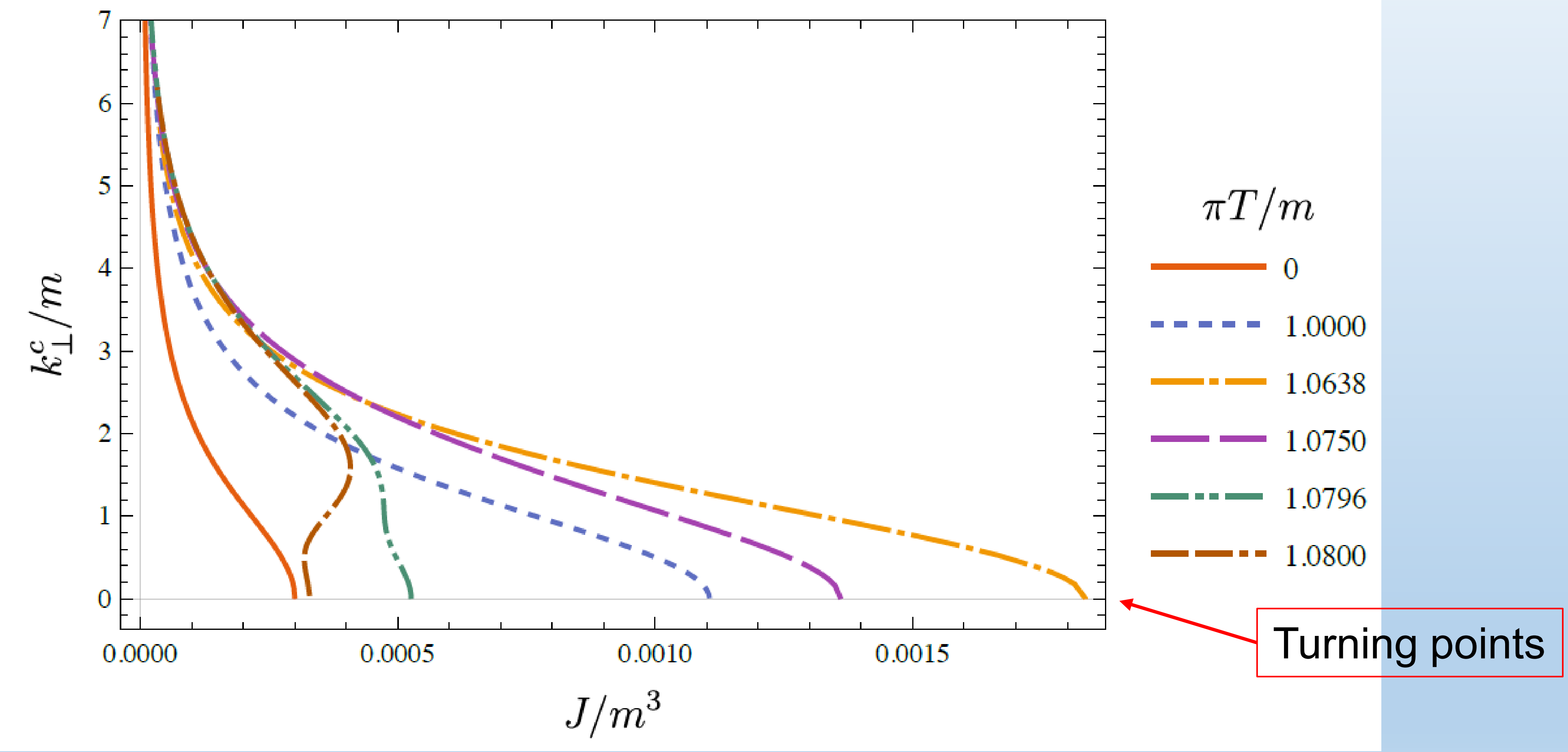
Considering inhomogeneous perturbations with finite $k_\perp$, the pure-imaginary mode moves to the lower-plane.

➡ The system is unstable with respect to homogeneous perturbations, but stable with respect to inhomogeneous perturbations.

➡ Inhomogeneous perturbations with critical momentum corresponds to static perturbations. This implies there exists an inhomogeneous solutions such that the current density is spatially modulated.

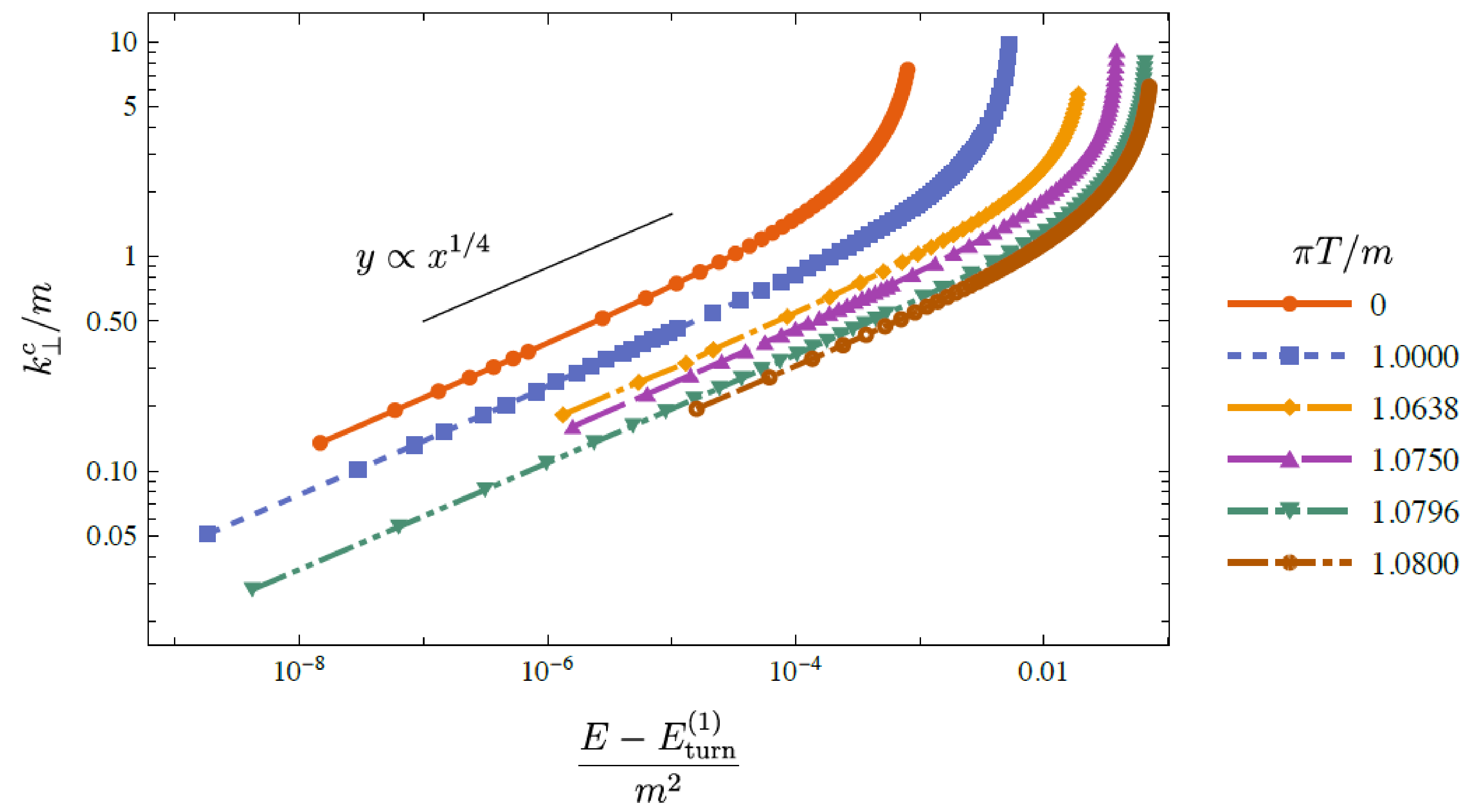
Inhomogeneous perturbations

Critical momentum vs current density at various temperature.



Inhomogeneous perturbations

Critical momentum vs electric field



$$\frac{k_\perp^c}{m} \approx c\left(\frac{E - E_{\text{turn}}^{(1)}}{m^2}\right)^p$$

$\pi T/m$	0	1.0000	1.0796
p	0.260	0.257	0.251

Critical momentum seems to behave as a power function of $E - E_{\text{turn}}$.

Summary

- Our system has a constant current flow and shows the nonlinear conductivity. (J is a multivalued function of E.)
- We study linear stability of the system near the multivalued region.
- One branch is dynamically stable but another branch is unstable with respect to a homogeneous perturbations.
- Unstable branch can be stable for inhomogeneous perturbations.
  ➡ It implies an inhomogeneous state with a spatially modulated current.
  ➡ holographic realization of current filaments.